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THE SOLUTION OF PROBLEMS OF THE THEORY OF ELASTICITY FOR AN ISOTROPIC PHYSICALLY NONLINEAR MATERIAL

Abstract: The paper presents a solution to physically nonlinear problems of the theory of elasticity for continuous isotropic bodies. The presented solution method is a synthesis of the boundary state method and the small parameter method. As a result of the expansion of the desired characteristics of the stress-strain state into power series, it becomes necessary to solve a number of linear problems in the theory of elasticity. The latter is provided by the boundary state method. The solution of problems and assessment of the accuracy of the results.

Key words: Physically nonlinear problems, honey boundary states, small parameter method, boundary value problems, isotropic bodies.

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Introduction

The solution of physically nonlinear problems reduces to nonlinear differential equations, the analytical solution of which can be obtained only in the simplest cases. Therefore, various approximate methods for solving physically nonlinear problems and related problems of plasticity are widespread. These methods are based on the linearization of differential equations and are reduced to solving problems of the theory of elasticity. Such methods include: the method of elastic solutions (method A. A. Ilyushin [1]), the method of variable parameters of elasticity, reducing the solution of nonlinear problems to the solution of a number of linear problems of the theory of elasticity for inhomogeneous bodies, the method of sequential loading (step method), based on summing n elastic problems when dividing the external load into n small values.

The aim of the work is to develop a method for constructing the fields of characteristics of a stress-

strain state for a homogeneous physically nonlinear isotropic body. The system of interconnected procedures meets its achievement: the correct formulation of the problem, the dimensionlessness (P-theorem), the choice of a solution method, and verification of results.

An effective tool for constructing elastic fields of isotropic and anisotropic bodies has been the modern energy method of boundary states (MGS) [2], which was initially oriented towards computer algebras. Its development in terms of connecting the perturbation method (MGSV) [3] allows you to effectively cope with the features of the physical plan for the environment.

The boundary state method is used to solve a wide class of problems in the theory of elasticity. Thermoelasticity problems were investigated, anisotropic problems were considered, for example, in [4] plane problems of the theory of elasticity for a doubly connected region were solved, and in [5] the

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proposed method for solving plane problems was generalized to the spatial case.

A number of works are devoted to solving boundary value problems of the theory of elasticity with the participation of mass forces [6-9]. The peculiarity of the solution is that the elastic field satisfies the given mass and surface forces at the same time.

Below is a methodology for constructing a solution to the spatial problem of physically nonlinear deformation of an isotropic medium.

$$\sigma_i = \frac{1}{\sqrt{2}} \sqrt{(\sigma_x - \sigma_y)^2 + (\sigma_y - \sigma_z)^2 + (\sigma_z - \sigma_x)^2 + 6(\tau_{xy}^2 + \tau_{yz}^2 + \tau_{zx}^2)} ;$$

strain rate

$$\varepsilon_i = \frac{\sqrt{2}}{3} \sqrt{(\varepsilon_x - \varepsilon_y)^2 + (\varepsilon_y - \varepsilon_z)^2 + (\varepsilon_z - \varepsilon_x)^2 + 3/2(\gamma_{xy}^2 + \gamma_{yz}^2 + \gamma_{zx}^2)} .$$

The same quantities expressed in terms of principal stresses:

$$\sigma_i = \frac{1}{\sqrt{2}} \sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2} ; \quad (1)$$

$$\varepsilon_i = \frac{\sqrt{2}}{3} \sqrt{(\varepsilon_1 - \varepsilon_2)^2 + (\varepsilon_2 - \varepsilon_3)^2 + (\varepsilon_3 - \varepsilon_1)^2} . \quad (2)$$

Statement of the problem and the mathematical model of its solution

A physically nonlinear medium from a homogeneous continuous isotropic material is investigated. In the physically nonlinear theory, as well as in the theory of plasticity, the following concepts are used [10]: stress intensity

The dependence of the stress intensity on the strain intensity is shown in Fig. 1. Curve 1 corresponds to a linear dependence, 2 - nonlinear.

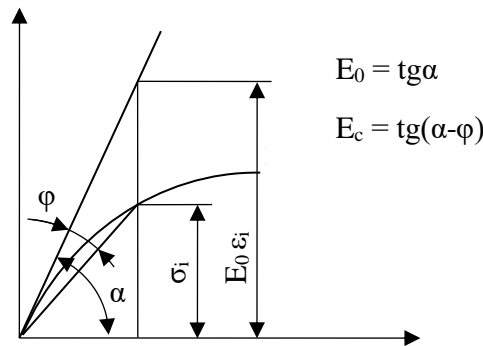


Рисунок 1 - Зависимость между интенсивностями напряжений и деформаций

On the image 1 E_0 – elastic modulus, E_c – secant modulus, moreover

$$E_c = \frac{\sigma_i}{\varepsilon_i} . \quad (3)$$

We introduce a small parameter β characterizing the deviation of the secant modulus from the elastic modulus:

$$E_c = E_0(1 - \beta) . \quad (4)$$

Let the material have a nonlinear tensile-compression diagram described by the relation

$$\sigma_i = A\varepsilon_i - B\varepsilon_i^k , \quad (5)$$

where A, B, k – are material constants determined from experiment. From (4) it follows

$$\beta = 1 - \frac{E_{\bar{n}}}{E_0} .$$

Substituting in the last equality the dependencies (3) and (5), we obtain

$$\beta = 1 - \frac{A}{E_0} - \frac{B}{E_0} \varepsilon_i^{k-1} . \quad (6)$$

The relationship between stress intensity and strain intensity does not depend on the type of stress state. From this it follows that the dependence $\sigma_i = f(\varepsilon_i)$ is the same for any combination of stresses and strains and can be determined from any experiment, for example, uniaxial tension, in which the main stresses and strains:

$$\sigma_1 = \sigma ; \sigma_2 = \sigma_3 = 0 ; \varepsilon_1 = \varepsilon ; \varepsilon_2 = \varepsilon_3 = -\nu\varepsilon ,$$

where ν – Poisson's ratio. Substituting these values in (1) and (2), we obtain

$$\sigma_i = \sigma ; \varepsilon_i = \frac{2(1+\nu)}{3} \varepsilon . \quad (7)$$

Having a dependence of $\sigma \sim \varepsilon$ under uniaxial tension, according to formulas (7), one can obtain a dependence of $\sigma_i \sim \varepsilon_i$. The maximum strain value is ε – the value known from experience on uniaxial

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tension, substituting it into the right one from formulas (7), is determined by ε_i , and then by (6) the small parameter β is calculated.

The state of the environment is subject to Hooke's law [11]:

$$\begin{aligned}\sigma_x &= \lambda\theta + 2\mu\varepsilon_x; \sigma_{xy} = 2\mu\varepsilon_{xy}; \\ \sigma_y &= \lambda\theta + 2\mu\varepsilon_y; \sigma_{yz} = 2\mu\varepsilon_{yz}; \\ \sigma_z &= \lambda\theta + 2\mu\varepsilon_z; \sigma_{xz} = 2\mu\varepsilon_{xz},\end{aligned}$$

where λ и μ – Lamé parameters; θ – volumetric deformation, $\theta = \varepsilon_x + \varepsilon_y + \varepsilon_z$.

If in Hooke's law instead of Young's module we use the secant module (4), then it will have the form:

$$\begin{aligned}\sigma_{xx} &= \lambda\theta + 2\mu\varepsilon_{xx} + 2/3\mu\theta\beta - 2\mu\varepsilon_{xx}\beta; \\ \sigma_{yy} &= \lambda\theta + 2\mu\varepsilon_{yy} + 2/3\mu\theta\beta - 2\mu\varepsilon_{yy}\beta; \\ \sigma_{zz} &= \lambda\theta + 2\mu\varepsilon_{zz} + 2/3\mu\theta\beta - 2\mu\varepsilon_{zz}\beta; \\ \tau_{xy} &= 2\mu\varepsilon_{xy} - 2\mu\varepsilon_{xy}\beta; \\ \tau_{yz} &= 2\mu\varepsilon_{yz} - 2\mu\varepsilon_{yz}\beta; \\ \tau_{xz} &= 2\mu\varepsilon_{xz} - 2\mu\varepsilon_{xz}\beta,\end{aligned}\quad (8)$$

where

$$\mu = \frac{E_0}{2(1+\nu)}; \lambda = \frac{E_0\nu}{(1+\nu)(1-2\nu)}.$$

This purpose allows us to describe the actual behavior of a physically nonlinear medium through the constants of a certain elastic medium and a small parameter β , the zero value of which corresponds to a linear isotropic medium.

We introduce the asymptotic series:

$$\begin{aligned}u_i &= \sum_{n=0}^{\infty} \beta^n u_i^{(n)}; \varepsilon_{ij} = \sum_{n=0}^{\infty} \beta^n \varepsilon_{ij}^{(n)}; \\ \theta &= \sum_{n=0}^{\infty} \beta^n \theta^{(n)}; \sigma_{ij} = \sum_{n=0}^{\infty} \beta^n \sigma_{ij}^{(n)}.\end{aligned}\quad (9)$$

Superscripts in parentheses, equal to the degrees of a small parameter, identify the number of the corresponding element in the asymptotic series.

Hooke's law (8) after replacing the summation and postulate variables with zero values for any formally non-existent decomposition element for which the index has a negative value ($n < 0$) leads to the corollary:

$$\begin{aligned}\sigma_{xx}^{(n)} &= \lambda\theta^{(n)} + 2\mu\varepsilon_{xx}^{(n)} + \tilde{\sigma}_{xx}^{(n)}; \\ \sigma_{yy}^{(n)} &= \lambda\theta^{(n)} + 2\mu\varepsilon_{yy}^{(n)} + \tilde{\sigma}_{yy}^{(n)}; \\ \sigma_{zz}^{(n)} &= \lambda\theta^{(n)} + 2\mu\varepsilon_{zz}^{(n)} + \tilde{\sigma}_{zz}^{(n)}; \\ \sigma_{xy}^{(n)} &= 2\mu\varepsilon_{xy}^{(n)} + \tilde{\sigma}_{xy}^{(n)}; \\ \sigma_{yz}^{(n)} &= 2\mu\varepsilon_{yz}^{(n)} + \tilde{\sigma}_{yz}^{(n)}; \\ \sigma_{xz}^{(n)} &= 2\mu\varepsilon_{xz}^{(n)} + \tilde{\sigma}_{xz}^{(n)}; \\ \tilde{\sigma}_{xx}^{(n)} &= 2/3\mu\theta^{(n-1)} - 2\mu\varepsilon_{xx}^{(n-1)}; \\ \tilde{\sigma}_{yy}^{(n)} &= 2/3\mu\theta^{(n-1)} - 2\mu\varepsilon_{yy}^{(n-1)}; \\ \tilde{\sigma}_{zz}^{(n)} &= 2/3\mu\theta^{(n-1)} - 2\mu\varepsilon_{zz}^{(n-1)}; \\ \tilde{\sigma}_{xy}^{(n)} &= -2\mu\varepsilon_{xy}^{(n-1)}; \\ \tilde{\sigma}_{yz}^{(n)} &= -2\mu\varepsilon_{yz}^{(n-1)}; \\ \tilde{\sigma}_{xz}^{(n)} &= -2\mu\varepsilon_{xz}^{(n-1)}.\end{aligned}$$

After redesignation (tensor-index form of record):

$$s_{ij}^{(n)} = \sigma_{ij}^{(n)} - \tilde{\sigma}_{ij}^{(n)},\quad (10)$$

we obtain for the decomposition elements the familiar form of the generalized Hooke law for an isotropic body:

$$s_{ij}^{(n)} = \lambda\theta^{(n)} + 2\mu\varepsilon_{ij}^{(n)}.\quad (11)$$

Cauchy's ratio is converted to a similar form:

$$\varepsilon_{ij}^{(n)} = \frac{1}{2}(u_{i,j}^{(n)} + u_{j,i}^{(n)}).\quad (12)$$

Denoting through X_i^0 volume forces and assuming the series to be known

$$X_i^0 = \sum_{n=0}^{\infty} \beta^n X_i^{0(n)},$$

we rewrite the equilibrium equations in the form

$$s_{ij,j}^{(n)} + X_i^{(n)} = 0; X_i^{(n)} = X_i^{0(n)} + \tilde{\sigma}_{ij,j}^{(n)}.\quad (13)$$

Relations (11) - (13) in shape correspond to the deformed state of an isotropic linearly elastic body.

Solution method

Any internal state of a linear isotropic elastostatic medium constitutes a set of displacements, strains, stresses $\xi = \{u_i, \varepsilon_{ij}, \sigma_{ij}\} \in \Xi$ coordinated by the defining relations. Their trace at the boundary ∂V of region V with a single external normal contains information about displacements and forces along the boundary $\gamma = \{u_i, p_i\} \in \Gamma$, $p_i = \sigma_{ij}n_j$ and corresponds to the boundary state. The spaces of possible internal and boundary states are Hilbert and isomorphic [2]: $\Xi \leftrightarrow \tilde{A}$.

Any correct problem reduces to an infinite system of linear algebraic equations

$$Qc = q,\quad (14)$$

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with respect to the vector of Fourier coefficients \mathbf{c} of the expansion of the desired state in a series along an orthonormal basis

$$\xi = \sum_l c_l \xi^{(l)}. \quad (15)$$

The Q matrix is structurally determined only by the type of boundary conditions (BC) and numerically through an orthonormal basis. In the first and second main tasks, matrix Q is the identity matrix. The vector of the right parts includes information about the specific content of the BC.

At each step, an infinite system of equations (14) is formulated in accordance with the BC of this iteration. In practice, it is quite realistic to consider BC only at $n=0$, solving only the main problem with $Q \equiv E$ in subsequent iterations and taking into account the corrections on the right-hand side caused by the appearance of fictitious volume forces in the ratios, which in the general case are not potential, but have a polynomial character. The general method of finding the internal state for a class of such forces is known [12].

Before performing iterations, the following actions are performed: on the basis of the general Papkovitch-Neiber solution and the basis of harmonic functions in $V \cup \partial V$, the bases of spaces Ξ and Γ are formed [2]; isomorphic orthonormal bases are constructed; members of $X_i^{0(n)}$ decomposition series for X_i^0 are established. Due to the independence of the initial basis from parameter β , the orthonormal basis is constructed exactly once and then used in each iteration.

At step $n=0$: state $\tilde{\xi}^{(0)}$ is sought due to volume forces $X_i^{0(n)}$; in real BC, a correction is made corresponding to this state, an infinite system of equations $Q\mathbf{c}^{(0)} = \mathbf{q}$ is formed; its solution and linear combination (15) give an internal state of $\xi^{(0)}$; its sum

with the state of the bulk forces prepares the initial approximation for ξ : $\xi = \xi^{(0)} + \tilde{\xi}^{(0)}$. According to the previous (10) formulas, the tensor $\tilde{\sigma}_{ij}^{(0)}$ is established.

At $n > 0$: the tensor $s_{ij}^{(n)} = \sigma_{ij}^{(n)} - \tilde{\sigma}_{ij}^{(n)}$ and the vector $X_i^{(n)}$ are constructed; state $\tilde{\xi}^{(n)}$ is sought due to volume forces $X_i^{(n)}$ in accordance with (13); in BC, the correction value from them is introduced and the first main problem for the system of equations (11) – (13) is solved; summing with the state of fictitious volume forces and adjusting the stress field in accordance with (10) $\sigma_{ij}^{(n)} = s_{ij}^{(n)} + \tilde{\sigma}_{ij}^{(n)}$, this additive can be included in the accumulated resulting state with a coefficient of β^n .

After performing a sufficient number of approximations, it is necessary to carry out the final substitution of the value β and go to dimensional values.

The solution of the problem

Testing of the proposed methodology was carried out on a rather simple first basic task for a cube-shaped body. After carrying out the dimensionlessness, an analogy of which is shown in [13], the body occupies the region $V = \{(x, y, z) | -1 \leq x, y, z \leq 1\}$ and the technical constants of the hypothetical isotropic material (4) – (6): $E_0 = 3$; $\mu = 0.5$; $A = 3$; $B = 2$; $k = 2$; $\varepsilon_i = 0.1$. Small parameter (6) $\beta = 1/15$.

Loaded along faces S_1 and S_2 by forces (Figure 2):

$$\{p_x, p_y, p_z\} = \begin{cases} \{1, 0, 0\}, (x, y, z) \in S_1 \\ \{-1, 0, 0\}, (x, y, z) \in S_2 \end{cases}$$

No mass forces: $X_i^0 = 0$.

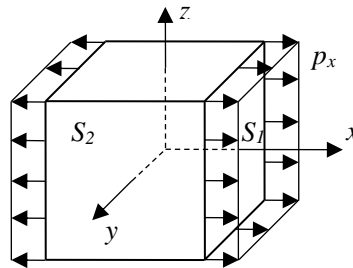


Figure 2 - Boundary conditions for the test problem

The application of the boundary-state method with perturbations allows us to consider an isotropic medium with dimensionless Young's modulus in tension $E_0 = 3$ and a Poisson's ratio of $\nu = 0.5$ at each iteration step.

Solution (9) is the series:

$$u = \sum_{n=0}^N \frac{1}{3} x \beta^n; \quad v = - \sum_{n=0}^N \frac{1}{6} y \beta^n;$$

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$$w = -\sum_{n=0}^N \frac{1}{6} z \beta^n ; \quad (16)$$

$$\sigma_x = 1; \sigma_y = \sigma_z = \sigma_{yz} = \sigma_{xz} = \sigma_{xy} = 0.$$

After substituting the small strain parameter (calculated by the Cauchy relations [8]) for $n = 3$, they are equal:

$$\varepsilon_x = 0.35713; \varepsilon_y = \varepsilon_z = -0.17856;$$

$$\varepsilon_{yz} = \varepsilon_{xz} = \varepsilon_{xy} = 0.$$

The error will be estimated by comparing the strains of the resulting state with the strains of the elastic state, where the secant modulus (4) $E_0 = E_c = 2.8$ is used as the elastic modulus. For the last state:

$$\varepsilon_x = 0.357143; \varepsilon_y = \varepsilon_z = -0.17857;$$

$$\varepsilon_{yz} = \varepsilon_{xz} = \varepsilon_{xy} = 0.$$

For deformations, the errors were: $\varepsilon_x - 0.36\%$; ε_y and $\varepsilon_z - 0.56\%$. Those three iterations to achieve satisfactory accuracy are sufficient.

We study the convergence of the obtained series with a significant increase in the small parameter. Now let $A = 3$; $B = 8$; $k = 2$; $\varepsilon_0 = 0.2$ and

$\beta = 0.53333$. Now the comparison must be carried out with the state at $E_0 = E_c = 1.4$. For this state of deformation:

$$\varepsilon_x = 0.714286; \varepsilon_y = \varepsilon_z = -0.357143;$$

$$\varepsilon_{yz} = \varepsilon_{xz} = \varepsilon_{xy} = 0.$$

After substituting a small parameter in series (16), deformations:

$$\text{at } n = 3: \varepsilon_x = 0.65649; \varepsilon_y = \varepsilon_z = -0.32824;$$

$$\varepsilon_{yz} = \varepsilon_{xz} = \varepsilon_{xy} = 0;$$

$$\text{at } n = 16: \varepsilon_x = 0.71427;$$

$$\varepsilon_y = \varepsilon_z = -0.357135; \varepsilon_{yz} = \varepsilon_{xz} = \varepsilon_{xy} = 0.$$

For the latter case, the errors were: $\varepsilon_x - 0.22\%$; ε_y and $\varepsilon_z - 0.22\%$. Accuracy is ensured by increasing the number of iterations.

The material considered earlier was incompressible ($\nu_0 = 0.5$). When using material with a non-0.5 Poisson's ratio, the accuracy of the calculations decreases. For example, for $E_0 = 3$; $\beta = 0.066666$; $n = 3$, the accuracy of the calculation of deformations depending on the Poisson's ratio is presented in table 1.

Table 1

Decision error analysis

	$\nu_0 = 0.4$	$\nu_0 = 0.3$	$\nu_0 = 0.2$	$\nu_0 = 0.1$	$\nu_0 = 0.05$
ε_x	0.44%	0.89%	1.33%	1.78%	2%
$\varepsilon_y, \varepsilon_z$	1.1%	2.96%	6.66%	17.77%	39.98%

It should be noted that the error is laid already at the first iteration and an increase in the number n does not lead to its decrease.

Conclusion

An analysis of the foregoing allows us to conclude that the MGSW has proven to be an effective means of writing out an explicit solution in physically nonlinear problems of mechanics for bodies made of materials in which the tensile-compression diagram is

described by a quadratic curve. The accuracy depends on the value of the Poisson's ratio, since Hooke's law (8) describes the dependence of strains on stresses in the theory of plasticity, in which, as you know, a Poisson's ratio of 0.5 is taken.

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