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MATHEMATICAL MODELING OF MAGNETOELASTIC VIBRATIONS OF A ROD IN A MAGNETIC FIELD

Abstract: When solving geometrically nonlinear problems of magnetoelasticity in the theory of plates and shells, it is difficult, in a general case, to evaluate the effect of nonlinearity in determining their stress state. To evaluate such a process, nonlinear oscillations of an isotropic rod of constant cross section under the influence of the electromagnetic Lorentz force are considered. The obtained estimates for the rod also characterize the qualitative side of the behavior of flexible plates and shells in a magnetic field.

Key words: rod, shell, magnetic field, magneto elasticity.

Language: English

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Introduction

In the mechanics of conjugate fields, an important place is occupied by the study of a continuous medium motion taking into account electromagnetic effects. Analysis of electromagnetic processes is possible only on the basis of a system of electrodynamic equations, together with material relationships. In recent decades, considerable attention in special literature has been devoted to the study of strain processes in electrically conductive bodies placed in an external constant magnetic field under the influence of force, thermal, and electromagnetic loads [1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19,20,21,22,23,24,25].

Interest in research in this area is associated with the importance of a quantitative study and assessment of the observed effects of the relationship of mechanical, thermal and electromagnetic processes and their practical application in various fields of modern technology in the development of new practices, as well as in the field of radio electronics, electrical engineering, modern measuring systems and etc. Most of the known works on elastic conductive body strain have been performed for a linearized system of equations. However, the solution of a number of applied problems, which include non-stationary problems of determining the stress state of flexible current-carrying anisotropic plates and shells, requires a more complete study of mechanical

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processes, including wave fields accompanying magnetoelastic interaction, based on a nonlinear model of magnetoelasticity and presents an urgent scientific problem.

I. STATEMENT OF THE PROBLEM. THE EQUATIONS OF MAGNETOELASTICITY.

Suppose that an electrically conductive body is in a magnetic field formed by an electric current in the body itself (self-magnetic field) and a source located at a distance from the body (external magnetic field). The body has finite electrical conductivity σ and does not have the property of spontaneous polarization and magnetization. Let's assume that, in the general case, there are no surface currents and foreign charges.

Magnetoelasticity equations for similar bodies in the Euler coordinates are presented in the form [3,9,10]:

equations of motion:

$$\frac{\partial t_{ki}}{\partial x_k} + \rho(F_i + F_i^{\wedge}) = \rho \frac{dV_i}{dt}, \quad (1)$$

$$\rho F_i^{\wedge} = \varepsilon_{ilm} J_l B_m + \rho_e E_i, \quad (2)$$

Maxwell equations

$$\varepsilon_{ijk} \frac{\partial H_k}{\partial x_j} = J_i + \frac{\partial D_i}{\partial t}, \quad \frac{\partial B_i}{\partial x_i} = 0, \quad (3)$$

$$\varepsilon_{ijk} \frac{\partial E_k}{\partial x_j} = -\frac{\partial B_i}{\partial t}, \quad \frac{\partial D_i}{\partial x_i} = \rho_e. \quad (4)$$

In relations (1)-(4) the following notation is introduced: t_{ij} - the components of the tensor of internal stresses; ρF_i - the vector components of volume mechanical forces; ρF_i^{\wedge} - the components of the vector of Lorentz volume forces; E_k, D_k, H_k, B_k, J_k - the components of the vectors of intensity and induction of the electric field, intensity and induction of the magnetic field, respectively; $J_k = J_k^* + \rho_e V_k$ - the components of the density vector of total current; J_k^* - conduction current density; $\rho_e V_k$ - the convective current density; ρ_e - the density of electric charges; ρ - the density of the substance in the current state; V_k - the components of the velocity vector; $\frac{d}{dt} = \frac{\partial}{\partial t} + V_k \frac{\partial}{\partial x_k}$ - the total time derivative.

It is necessary to add kinematic equations for the processes of electrical and thermal conductivity and the governing equations to the equations of magnetoelasticity (1)-(4). Later, we will neglect the temperature effects, and take the kinematic equations for the processes of electrical conductivity in the form of Ohm's law:

$$J_i = \sigma \left(E_i + \varepsilon_{ijk} \frac{\partial u_j}{\partial t} B_k \right) + \rho_e \frac{\partial u_i}{\partial t}, \quad (5)$$

and

$$D_i = \varepsilon E_i, \quad B_i = \mu H_i, \quad (6)$$

where ε is the coefficient of electrical permeability, μ is the coefficient of magnetic permeability ($\mu = \mu_0, \varepsilon = \varepsilon_0$).

When setting the boundary value problems, it is necessary to formulate the boundary conditions for mechanical and electromagnetic characteristics. In spatial variables for the full stress tensor, it can be written:

$$v_k(t_{ki} + \tau_{ki}) / S_1 = \left[P_i + v_k \tau_{ki}^{(c)} \right] / S_1. \quad (7)$$

Here τ_{ki} is the Maxwell tensor

$$\tau_{ki} = E_i D_k + H_i B_k - 1/2 \delta_{ki} (E_j D_j + H_j B_j) \quad (8)$$

$\tau_{ki}^{(c)}$ - is the Maxwell tensor in vacuum; P_i - the components of surface forces related to the site dimensions in a strained state; v_k - the components of the unit normal vector to the strained boundary of the body; S_1 - the part of the body boundary on which the boundary conditions in stresses are set.

II. NUMERICAL EXAMPLE. ANALYSIS OF RESULTS.

Consider a rectilinear rod made of aluminum, l long with a hinged fixing in its ends. We believe that the flexible rod is in a constant external magnetic field and serves as a conductor of electric current, supplied to the ends of the rod from an external source and is a function of time t . As a result of current interaction with the magnetic field, the Lorentz volume forces arise in the rod [3]:

$$\rho \vec{f} = \vec{J} \times \vec{B} \quad (9)$$

The current density is set by the formula

$$\vec{J} = -J_{10} \cdot \sin \omega t \vec{i}, \quad (10)$$

and the magnetic induction vector is taken as constant

$$\vec{B} = B_0 \cdot \vec{j} \quad (11)$$

where ω - is the circular frequency. In this case, the ponderomotive force is

$$\rho \vec{f} = J_{10} \cdot B_0 \cdot \sin \omega t \vec{k} \quad (12)$$

The equation of the rod transverse bending according to the balance of forces acting on the element along the axis z takes the form

$$\begin{aligned} \frac{Eh^3}{12} \frac{\partial^4 w}{\partial x^4} - h \sigma_x \frac{\partial^2 w}{\partial x^2} + \rho h \frac{\partial^2 w}{\partial t^2} = \\ = J_{10} \cdot B_0 \cdot \sin \omega t, \end{aligned} \quad (13)$$

where σ_x - is the membrane part of the longitudinal normal stress.

Boundary conditions are:

$$w = 0, \quad \partial^2 w / \partial x^2 = 0 \quad \text{at } x = 0, \quad x = l \quad (14)$$

Initial conditions are:

$$w = 0, \quad \dot{w} = 0 \quad \text{at } t = 0 \quad (15)$$

For the case where only the transverse load acts ($\sigma_x = 0$, linear case) equation (13) takes the form

$$\frac{Eh^3}{12} \frac{\partial^4 w}{\partial x^4} + \rho h \frac{\partial^2 w}{\partial t^2} = J_{10} \cdot B_0 \cdot \sin \omega t, \quad (16)$$

Equation (13) is a differential equation of forced bending vibrations of a rod of constant cross section. Consider the physical meaning of various terms of equation (13). Their signs depend on the chosen sign rule, and do not have a special physical meaning. The first term of equation is the deflection resistance, calculated as a variation of transverse force, the moment of which balances the variation of the bending moment, which occurs due to a change in curvature, i.e. we have the bending resistance to deflection proportional to the bending rigidity of the rod.

The second term is the transverse component caused by the curvature of a certain axial force N_x . If the force N_x does not depend on deflection due to the axial load applied at the ends and so that it remains constant under bending, then the second term is linear with respect to w . Axial force N_x can also be caused by deflection. This happens if the rod supports prevent the ends of the rod from moving towards each other. Then, if the rod is bent by transverse forces, the axial line will extend, and since it bends, and therefore becomes longer, than it was originally, the supports will create tensile force N_x acting on the rod, which will increase in proportion to the square of deflection. The second term in this case will increase in proportion to the third degree of deflection, and the equation will become non-linear with respect to w .

The third term presents the action of inertial volume load. The fourth term of equation is a transverse load, tending to cause deflection. Representing the electromagnetic load as

$$J_0 \cdot B_0 \sin \omega t \sin \pi x / l, \quad (17)$$

a solution to the boundary value problem (13) - (15) is sought in the form

$$w(x, t) = w_1 \sin \omega t \sin \pi x / l, \quad (18)$$

where w_1 – is the deflection in the middle of the rod span. Before proceeding to the solution of the problem, determine the normal stress σ_x . Let Δl – be the difference between the lengths of the curved and non-curved axes of the rod. Then

$$\sigma_x = E \frac{\Delta l}{l} = \frac{E}{l} \int_0^l \sqrt{1 + \left(\frac{\partial w}{\partial x} \right)^2} - 1 \, dx \approx \quad (19)$$

$$\approx \frac{E}{2l} \int_0^l \left(\frac{\partial w}{\partial x} \right)^2 dx = \frac{\pi^2 E}{4l^2} w_1^2 \sin^2 \omega t.$$

Substituting expression (18) and (19) into equation (13), after dividing by $\sin \frac{\pi x}{l}$, we have

$$\frac{Eh^3}{12} \left(\frac{\pi}{l} \right)^4 w_1 \sin \omega t + Eh \left(\frac{\pi}{l} \right)^4 w_1^3 \sin \omega t - \quad (20)$$

$$- \rho h \omega^3 w_1 \sin \omega t = J_0 B_0 \sin \omega t.$$

Given that

$$\sin^3 \omega t = \frac{3}{4} \sin \omega t - \frac{1}{4} \sin 3\omega t$$

and collecting the coefficients at $\sin \omega t$, we obtain

an approximate relation with respect to w_1/h in the form

$$\frac{w_1}{h} + 9 \left(\frac{w_1}{h} \right)^3 = \frac{12l^4}{Eh^4 \pi^4} \left(J_0 B_0 + \rho h^2 \omega^2 \frac{w_1}{h} \right). \quad (21)$$

The first term of relation (21) characterizes the load resistance due to bending; the second term characterizes the resistance due to the force action

$$\sigma_x h \frac{\partial^2 w}{\partial x^2}.$$

In Fig. 1, the dashed line shows the resistance due to the bending only, the solid line shows the total resistance, where P – is the right-hand side of expression (21). It can be seen from Fig. 1 that the elementary linear theory gives a good approximation until the deflection is small, say, of an order $w_1 \leq 0.3h$ compared to the height of the cross section. For large deflections, the part of the load

corresponding to the second term $\sigma_x h \frac{\partial^2 w}{\partial x^2}$ grows

rapidly and therefore must be taken into account.

By resolving relation (21) with respect to the square of the circular frequency, we have

$$\omega^2 = \frac{Eh^2 \pi^4}{12 \rho l^4} \left[1 + 9 \left(\frac{w_1}{h} \right)^2 \right] - \frac{J_0 B_0}{\rho h^2} \left(\frac{h}{w_1} \right). \quad (22)$$

To implement the oscillatory process, it is necessary for the right-hand side of relation (22) to be positive. Fulfillment of this requirement allows one to determine the limits of variation of the current density depending on w_1/h at known B_0 . Failure to do so

leads to the fact that part of the frequencies is equal to zero or to an imaginary value. The amplitudes of the corresponding oscillations will increase unlimitedly.

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This is due to reaching or exceeding the critical value according to Euler, and the rod loses stability.

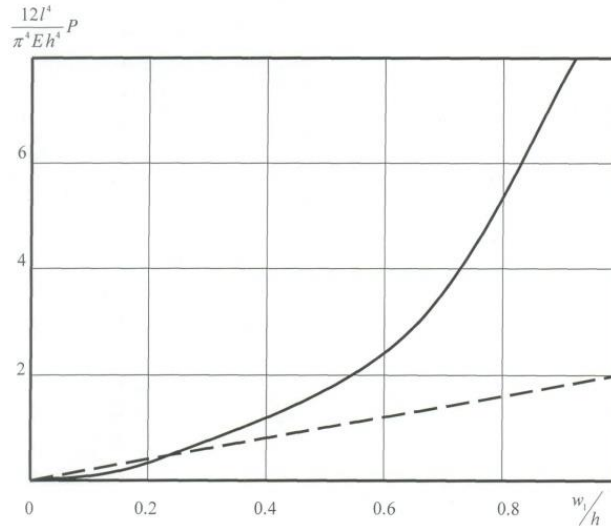


Fig. 1. Deflection resistance.

Figure 2 shows the dependencies J_0 on relationship w_1/h in linear and nonlinear cases. The dashed line indicates the linear case, the solid line indicates the nonlinear case at $B_0 = 0.5T$ and at the following parameters of the rod:

$$l = 0.5 \text{ m}; \quad h = 2 \times 10^{-3} \text{ m};$$

$$E = 7.1 \times 10^{10} \frac{\text{N}}{\text{m}^2}; \quad \rho = 2670 \frac{\text{kg}}{\text{m}^3}.$$

From the graphs it follows that for the corresponding values of w_1/h of oscillatory process, the component of the current density vector J_0 must take the values that are below the given lines.

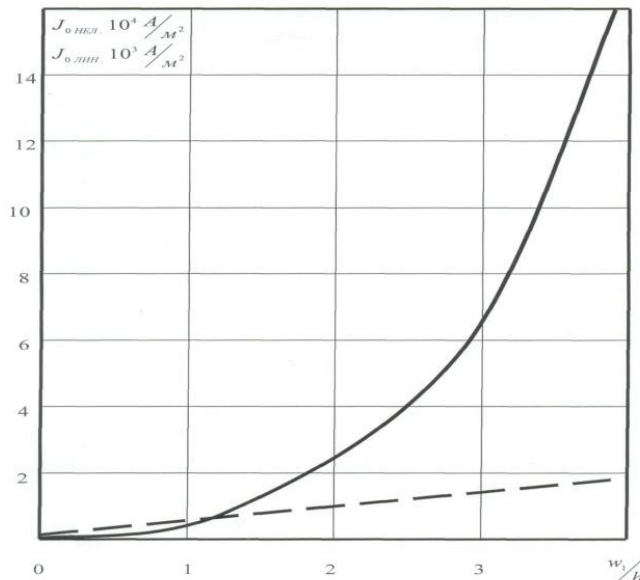


Fig. 2. Comparison of solutions in linear and nonlinear cases

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From the same graphs it follows that, given the appropriate values of J_0 , it is possible to determine the values for the corresponding relations w_1/h .

Note that everything said above regarding the considered rod characterizes qualitatively the behavior of flexible plates and shells located in an electromagnetic field.

III. CONCLUSIONS.

When a material body moves in an electromagnetic field, force interaction and energy exchange occur between the body and the field, due to

conduction currents and body polarization and magnetization phenomena. If to ignore the body polarization and magnetization, and consider only the conduction currents, then the force interaction of the body and the field occurs only due to the Lorentz forces, and the energy exchange is caused by the Joule heat only. To assess the influence of nonlinearity in determining the stress-strain state of flexible current-carrying plates and shells, nonlinear oscillations of an isotropic rod of constant cross section under the Lorentz electromagnetic force are considered. The obtained estimates for the rod also characterize the qualitative side of flexible plates and shells behavior in a magnetic field.

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