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STRESS-STRAIN STATE OF CURRENT-CARRYING SHELLS IN A MAGNETIC FIELD

Abstract: *A strain in a flexible current-carrying orthotropic cone with orthotropic conductivity is considered in the paper, when it is exposed to an external magnetic field and an external electric current. The effect of accounting for the conicity when determining the stress state of current-carrying orthotropic shells in a geometrically nonlinear statement is studied. It was revealed that the interaction of magnetic induction and shear force causes the appearance of extreme values of shell stresses.*

Key words: *shell, magnetic field, magneto elasticity.*

Language: *English*

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Introduction

In the mechanics of conjugate fields, an important place is occupied by the study of a continuous medium motion taking into account electromagnetic effects. Analysis of electromagnetic processes is possible only on the basis of a system of electrodynamic equations, together with material relationships.In recent decades, considerable attention in special literature has been devoted to the study of strain processes in electrically conductive bodies placed in an external constant magnetic field under the influence of force, thermal, and electromagnetic loads [1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19,20,21,22,23,24,25].

The interaction mechanism of an elastic medium with an electromagnetic field is diverse and depends on geometrical characteristics and physical properties of the body in question. In particular, this mechanism receives some specific features when the problem is considered with respect to thin plates and shells having orthotropic electrical conductivity. In most cases, the interaction of an electromagnetic field with an elastic body occurs in the presence of an external electric current. In this case, the problem is reduced to the problem of electromagnetoelasticity. However, the problems associated with the consideration of external currents are generally quite complex; they could be significantly simplified in the case of thin bodies subject to small changes in shape under strain.

I. STATEMENT OF THE PROBLEM. THE EQUATIONS OF MAGNETOELASTICITY.

Assuming that an external magnetic field acts on the body, the magnetoelasticity equations in the Lagrangian variables in the region occupied by the body (internal problem)can be represented in the form [2 - 4, 10]:

$$rot\,\vec{E} = -\frac{\partial \vec{B}}{\partial t};\; rot\,\vec{H} = \vec{J} + \vec{J}_{cm};$$

$$div\,\vec{B} = 0,\; div\,\vec{D} = 0; \qquad (1)$$

$$\rho\frac{\partial\vec{\upsilon}}{\partial t}=\rho(\vec{f}+\vec{f}^{\wedge})+div\hat{\sigma} \qquad (2)$$

where $\vec{E}$ – electric field strength; $\vec{H}$ – magnetic field strength; $\vec{B}$ – magnetic induction; $\vec{D}$ – electrical induction; $\vec{J}_{cm}$ – a density of foreign current, $\vec{f}$ – a volume mechanical force, $\vec{f}^{\wedge}$ – a Lorentz volume force, $\vec{J}$ – a density of current, ρ – is the density of the material; $\vec{\upsilon}$ – rate of deformation; $\hat{\sigma}$ – an internal stress tensor.

Geometrical and mechanical characteristics of the body are supposed such that a version of geometrically nonlinear theory of thin shells in quadratic approximation is applicable to describe the strain process. For the considered case of quadratic nonlinearity [2-4], the strains and shears are considered small in comparison with the rotation angles of the element, and the angles are significantly less than unity. Also suppose that the electromagnetic hypotheses hold with respect to the electric field $\vec{E}$ and magnetic field $\vec{H}$ [1]. Elastic properties of the shell material correspond to an orthotropic body, to the basic directions, which coincide in elasticity with the directions of corresponding coordinate lines; electromagnetic properties of the material are characterized by tensors of electrical conductivity σ_{ij}, magnetic permeability μ_{ij} and dielectric constant ε_{ij} $(i,j=1,2,3)$. Moreover, based on the crystal physics [2], it is supposed that the tensors σ_{ij}, ε_{ij} and μ_{ij} take a diagonal form for the class of conducting orthotropic media with a rhombic crystal structure. The system of equations of magnetoelasticity must be closed by relations connecting the vectors of intensity and induction of electromagnetic field, and by Ohm's law, which determines the density of conduction current in a moving medium. If an orthotropic body is linear with respect to magnetic and electrical properties, then the governing equations for electro-magnetic field characteristics and the kinematic equations for electrical conductivity, and the Lorentz force expressions, taking into account the external current $\vec{J}_{cm}$ in the Lagrange variables, are written in the form [10]:

$$\vec{B}=\mu_{ij}\vec{H},\quad \vec{D}=\varepsilon_{ij}\vec{E}, \qquad (3)$$

$$\vec{J}=\sigma_{ij}\Gamma F^{T}F^{-1}\left[\vec{J}_{cm}+\vec{E}+\vec{\upsilon}\times\vec{B}\right], \qquad (4)$$

$$\rho\vec{f}^{\wedge}=\Gamma^{-1}F^{-1}\left[\vec{J}_{cm}\times\vec{B}+\sigma_{ij}\left(\vec{E}+\vec{\upsilon}\times\vec{B}\right)\times\vec{B}\right] \qquad (5)$$

Here $\sigma_{ij},\varepsilon_{ij},\mu_{ij}$ are the tensors of electrical conductivity, dielectric and magnetic permittivities of linear current-carrying anisotropic body $(i,j=1,2,3)$ respectively. For homogeneous anisotropic media, they are symmetric second-rank tensors. Thus, (1), (2) with relations (3) - (5) make up a closed system of nonlinear equations of magnetoelasticity of current-carrying orthotropic bodies with orthotropy of conductive properties. The coordinate surface in the strain-free state refers to a curvilinear orthogonal coordinate system S and θ, where S – is the arc length of the generatrix (meridian), counted out from some fixed point, θ – is the central angle in a parallel circle, counted out from the selected plane. Coordinate lines $s=const$ and $\theta=const$ are the lines of the principal curvatures of coordinate surface. Choosing coordinate ζ normal to the coordinate surface of revolution, the shell refers to the spatial coordinate system s,θ,ζ. Suppose that the magnetic induction vector on the surface of conical shell and the surface mechanical forces are known. The coupled resolving system of magnetoelasticity equations of a thin current-carrying orthotropic shell is taken in the form presented in [15].

II. METHODS FOR SOLVING NONLINEAR PROBLEMS.

The method for solving the nonlinear problem of the magnetoelasticity of shells is based on the consistent use of the Newmark scheme, the quasilinearization method, and the discrete orthogonalization method [2 - 4, 7-17]. To separate the variables from the time coordinate, we apply the implicit Newmark scheme, with which the nonlinear boundary value problem reduces to a sequence of nonlinear one-dimensional boundary value problems at each time step. The next step in solving the sequence of nonlinear boundary value problems of magnetoelasticity is based on the application of the quasilinearization method, with which the nonlinear boundary value problem reduces to a sequence of linear boundary value problems. Then each of the linear boundary value problems of the sequence on the corresponding time interval is solved numerically with the help of the stable method of discrete orthogonalization.

III. . NUMERICAL EXAMPLE. ANALYSIS OF RESULTS.

Let us investigate the stress-strain state of a flexible orthotropic conical shell made of boron aluminum of constant thickness $h=5\cdot10^{-4}\ m$, under mechanical stress $P_{\zeta}=5\cdot10^{3}\sin\omega t\ N/m^{2}$. The shell is in an external magnetic field $B_{S0}=0.1T$ and is applied by the external electrical current of $J_{\theta CT}=-5\cdot10^{4}\sin\omega t\ \ A/m^{2}$, density. The shell has a finite orthotropic conductivity $\sigma(\sigma_1,\sigma_2,\sigma_3)$. Note that in this case the anisotropy of specific electrical resistivity is $\eta_3/\eta_1=2.27$. It is considered that an external electric current in an undisturbed state is uniformly distributed over the shell, i.e. the intensity of external current does not depend on coordinates. The effect of the conicity angle on the stress-strain state of an orthotropic conical shell is studied.

In this case, the combined effect on the shell loading, the ponderomotive force consisting of Lorentz forces and mechanical.

The boundary conditions are

$s = s_0 = 0: u = 0,\ Q_S = -200,\ M_S = 0,\ B_\zeta = 0.3\sin\omega t\, T;$

$s = s_N = 0.4m:\ w = 0,\quad \theta_S = 0,\quad B_\zeta = 0.$

The initial conditions are

$$\vec{N}(s,t)\big|_{t=0} = 0,\quad \dot{u}(s,t)\big|_{t=0} = 0,\quad \dot{w}(s,t)\big|_{t=0} = 0$$

The parameters of the shell and the material are:

$s_0 = 0,\ s_N = 0{,}4m,\ h = 5\cdot 10^{-4}\, m,$

$r_0 = 0.4\,m\ \ r = r_0 + s\cos\varphi;\quad \omega = 314.16\sec^{-1},$

$\varphi = \pi/15,\quad \rho = 2300\,kg/m^3,\ B_s^+ = B_s^- = 0.5\,T$.,

$\mu = 1.256\cdot 10^{-6}\, H/m,\quad J_{\theta cm} = -5\cdot 10^5 \sin\omega t\, A/m^2,$

$\sigma_1 = 0.454\cdot 10^8 (\Omega\times m)^{-1},\quad \sigma_2 = 0.454\cdot 10^8 (\Omega\times m)^{-1},$

$\sigma_3 = 0.200\cdot 10^8 (\Omega\times m)^{-1},\quad \nu_S = 0.262,\ \nu_\theta = 0.320,$

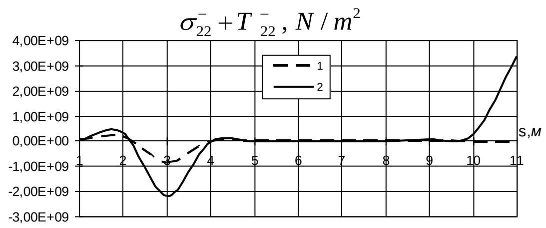
$P_\zeta = 5\cdot 10^3 \sin\omega t\, N/m^2,\ B_{S0} = 0.1\,T$

$e_S = 22.9\cdot 10^{10}\, N/m^2,\ e_\theta = 10.7\cdot 10^{10}\, N/m^2$

The solution is found in the time interval $\tau = 0 \div 10^{-2}\sec$ for the integration time step is chosen to be $\Delta t = 1\cdot 10^{-3}\sec$. Maximum values obtained at time step $t = 5\cdot 10^{-3}\sec$.

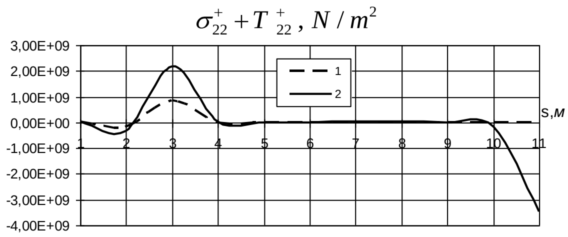
Figures 1 and 2 show the stress distributions $\sigma_{22}^- + T_{22}^-$ and $\sigma_{22}^+ + T_{22}^+$ along the shell meridian ***s*** at time $t = 5\cdot 10^{-3} s$, along the shell inner and outer surfaces at various angles: 1 - $\varphi = 5{,}8^0$; 2 - $\varphi = 5{,}7^0$.

When taking into account the influence of the conicity angle, the stress of conical shell was considered as a sum of mechanical stresses and Maxwell stresses, i.e. the total stress state was taken into account. Studying the graphs, it can be seen that at $\varphi = 5{,}7^0$ in point $s = 0{,}40\ m$, the stresses increase significantly. A sharp change in stress at $\varphi = 5{,}8^0$; $\varphi = 5{,}7^0$; in points $0{,}04\ m$, $0{,}08\ m$, $0{,}36\ m$, $0{,}40\ m$ is explained by the influence of boundary conditions. A shear force Q_s and a normal component of the magnetic field induction are applied at the left end of the cone $s = 0$.



$1 - \varphi = 5{,}8^0;\ \ 2 - \varphi = 5{,}7^0.$

Fig. 1. Stress distribution $\sigma_{22}^- + T_{22}^-$ along the shell meridian s, at time $t = 5\cdot 10^{-3} s$, on the shell inner surfaces at various angles φ.



$1 - \varphi = 5{,}8^0;\ \ 2 - \varphi = 5{,}7^0.$

Fig. 2. Stress distribution $\sigma_{22}^+ + T_{22}^+$ along the shell meridian s, at time $t = 5\cdot 10^{-3} s$, on the shell inner surfaces at various angles φ.

IV. CONCLUSION.

Coupled problems of magneto-elasticity were considered in the paper for a flexible orthotropic shell, taking into account the orthotropy of conductive properties. The results of numerical example were presented. Consideration was given to the effect of conicity on the nonlinear behavior of orthotropic shell. It was revealed that the angular opening of cone, equal to six degrees, turned out to be critical for the considered geometrically nonlinear shell under selected loads. Further decrease in angle ($\varphi = \pi/30$) led to a loss of stability of the shell.

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