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NONLOCAL PROBLEM FOR A HYPERBOLIC EQUATION DEGENERATING INSIDE A DOMAIN WITH A SINGULAR COEFFICIENT

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Introduction

1. Statement of the problem G.

Consider the equation

$$-|y|^m u_{xx} + u_{yy} - \frac{m}{2y} u_y = 0, m > 0. \quad (1)$$

Let Ω be a finite simply connected domain of the plane of independent variables x, y, bounded by the characteristics

$$\left.\begin{matrix} AC_1 \\ BC_1 \end{matrix}\right\}: \rightleftarrows x \mp \frac{2}{m+2} y^{\frac{m+2}{2}} = \mp 1, y > 0,$$

$$\left.\begin{matrix} BC_1 \\ BC_2 \end{matrix}\right\}: x \mp \frac{2}{m+2} (-y)^{\frac{m+2}{2}} = \mp 1, y < 0,$$

equation (1).

Task G. Find a regular solution in the field Ω

$$u(x,xy) = \begin{cases} u_1(x,xy), xx(x,xy) \in \Omega_1 = \Omega \cap \{y > 0\}, \\ u_2(x,xy), xx(x,xy) \in \Omega_2 = \Omega \cap \{y < 0\}. \end{cases}$$

equations (1) from the class $C(\bar{\Omega}_1 \cup \bar{\Omega}_2) \cap C^2(\Omega \backslash AB)$ satisfying the boundary conditions

$$u_j[\theta^{(j)}(x)] = \mu_1 u_j[\theta^{(j)}_{k_1}(x)] + \mu_2 u_j[\theta^{(j)}_{k_2}(x)] + $$
$$+ \frac{1}{2}\mu_1 u_j(p_1(x),0) - \frac{1}{2}\mu_2 u_j(p_2(x),0) +$$
$$\delta_j(x), \forall x \in I = AB \quad (2)$$

here, $j = 1$ corresponds to the area Ω_1, and $j = 2$ to the area Ω_2, $p_1(x) = a_1 + b_1 x, p_2(x) = a_2 + b_2 x$, where $a_i = \frac{2}{k_i+1}, b_i = \frac{k_i-1}{k_i+1}, i = 1,2$ and the pairing conditions

$$\lim_{y \to +0} u_1(x,y) = c \lim_{y \to -0} u_2(x,y), \forall x \in \bar{I} \quad (3)$$

$$\lim_{y \to +0} y^{-\frac{m}{2}} \frac{\partial u_1}{\partial y} = \rho(x) \lim_{y \to -0} (-y)^{-\frac{m}{2}} \frac{\partial u_2}{\partial y} + \lambda(x), \forall x \in I \quad (4)$$

where $\theta^{(j)}(x)(\theta^{(j)}_{k_1}(x), \theta^{(j)}_{k_2}(x))$

$$\theta^{(j)}(x_0) = \frac{1+x_0}{2} + (-i)^{j-1}\left[\frac{(m+2)(1-x_0)}{4}\right]^{\frac{2}{m+2}},$$

$$\theta^{(j)}_{k_1}(x_0) = \frac{1+k_1 x_0}{1+k_1} + (-i)^{j-1}\left[\frac{(m+2)(1-x_0)}{2(k_1+1)}\right]^{\frac{2}{m+2}},$$

$$\theta^{(j)}_{k_2}(x_0) = \frac{1+k_2 x_0}{1+k_2} +$$
$$+(-i)^{j-1}\left[\frac{(m+2)(1-x_0)}{2(k_2+1)}\right]^{\frac{2}{m+2}}$$

affix of the intersection point of the characteristic BC_j (curve $x + [2k_j/(m+2)]|y|^{(\tau+2)/2} = 1$, lying inside the area Ω_j) with a characteristic coming out of the point $M(x_0, 0) \in I$; $c = const$; $\mu_1, \mu_2 = const$; $\delta_j(x), \rho(x), \lambda(x)$ set functions from the class $C^2(\bar{I}) \cap C^3(I)$, where

$$\tau(1) = \tau'(1) = \tau''(1) = 0 \quad (5)$$

$$\rho(x) - c \neq 0, k_1 > k_2 > 1, \delta_j^{(n)}(1) = 0,$$

$$\lambda^{(n)}(1) = 0, n = 0,1,2.$$

1. Study of problem G.

The theorem. Task G when conditions are met

$$\lambda_1 + \lambda_2 < 1, \quad (6)$$

where $\lambda_k = \frac{b_k \mu_k}{b_1\mu_1 + b_2\mu_2 - 1} > 0, k = 1,2$ uniquely solvable.

Proof. 1. Consider the boundary condition (2). In the area Ω_1 the solution of the modified Cauchy problem [1] satisfying the conditions:

$$u_1(x, +0) = \tau_1(x), x \in \bar{I};$$

$$\lim_{y\to+0} y^{-\frac{m}{2}} \frac{\partial u_1}{\partial y} = \nu_1(x), x \in I \quad (7)$$

is given by the Dalembert formula [2]:

$$u(x,y) = \frac{\tau\left(x - \frac{2}{m+2}(-y)^{(m+2)/2}\right) + \tau\left(x + \frac{2}{m+2}(-y)^{(m+2)/2}\right)}{2} - \frac{(-y)^{(m+2)/2}}{m+2}\int_{-1}^{1} \nu\left[x + \frac{2t}{m+2}(-y)^{(m+2)/2}\right] dt, \quad (8)$$

From here it is easy to calculate that

$$u_1[\theta^{(1)}(x)] = \frac{1}{2}[\tau_1(x) + \tau_1(1)] - \frac{1}{2}\int_{-1}^{x} \nu_1(t)dt, \quad (9)$$

$$u_1[\theta_{k_1}^{(1)}(x)] = \frac{1}{2}[\tau_1(a_1 + b_1x) + \tau_1(x)] - \frac{1}{2}\int_x^{a_1+b_1x} \nu_1(t)dt, \quad (10)$$

$$u_1[\theta_{k_2}^{(1)}(x)] = \frac{1}{2}[\tau_1(a_2 + b_2x) + \tau_1(x)] - \frac{1}{2}\int_x^{a_2+b_2x} \nu_1(t)dt. \quad (11)$$

Now expressions (9)-(11), substituting into the boundary conditions (2), we obtain

$$\tau_1(x) - \int_x^1 \nu_1(t)dt = \mu_1\tau_1(x) + \mu_2\tau_1(x) - \mu_1\int_x^{a_1+b_1x} \nu_1(t)dt + \mu_2\int_x^{a_2+b_2x} \nu_1(t)dt + 2\delta_1(x). \quad (12)$$

Relation (12) is the first functional relation between unknown functions $\tau_1(x)$ and $\nu_1(x)$ [3], brought to the axis $y = 0$ from the area Ω_1.

2. Now consider the boundary condition (2) in the area Ω_2 using the solution [2] of the modified Cauchy problem satisfying the conditions:

$$u_2(x, -0) = \tau_2(x), x \in \bar{I};$$

$$\lim_{y\to-0} (-y)^{-\frac{m}{2}} \frac{\partial u_2}{\partial y} = \nu_2(x), x \in I \quad (13)$$

it is easy to calculate that

$$u_2[\theta^{(2)}(x)] = \frac{1}{2}[\tau_2(x) + \tau_2(1)] - \frac{1}{2}\int_{-1}^{x} \nu_2(t)dt, \quad (14)$$

$$u_2[\theta_{k_1}^{(2)}(x)] = \frac{1}{2}[\tau_2(a_1 + b_1x) + \tau_2(x)] - \frac{1}{2}\int_x^{a_1+b_1x} \nu_2(t)dt, \quad (15)$$

$$u_2[\theta_{k_2}^{(2)}(x)] = \frac{1}{2}[\tau_2(a_2 + b_2x) + \tau_2(x)] - \frac{1}{2}\int_x^{a_2+b_2x} \nu_2(t)dt. \quad (16)$$

Expressions (14)-(16), substituting into the boundary conditions (2), we obtain$\tau_2(x) - \int_x^1 \nu_2(t)dt = \mu_1\tau_2(x) + \mu_2\tau_2(x) - \mu_1\int_x^{a_1+b_1x} \nu_2(t)dt + \mu_2\int_x^{a_2+b_2x} \nu_2(t)dt + 2\delta_2(x).$ (17)

Relation (17) is the second functional relation between unknown functions $\tau_2(x)$ and $\nu_2(x)$, brought to the axis $y = 0$ from the area Ω_2.

From (12) and (17) according to the conditions of conjugation (3), (4), i.e. taking into account the equalities: $\tau_1(x) = c\tau_2(x)$, $\nu_1(x) = \rho(x)\nu_2(x) + \lambda(x)$ excluding $\tau_2(x)$ from (12), we obtain the following integral equation with respect to an unknown function $\nu_2(x)$:

$$\int_x^1 (\rho(t) - c)\nu_2(t)dt = \mu_1\int_x^{a_1+b_1x} (\rho(t) - c)\nu_2(t)dt + \mu_2\int_x^{a_2+b_2} (\rho(t) - c)\nu_2(t)dt + f(t) \quad (18)$$

where $f(x) = 2\delta_1(x) - 2c\delta_2(x) + \int_x^1 \lambda(t)dt + \mu_1\int_x^{a_1+b_1x} \lambda(t)dt + \mu_2\int_x^{a_2+b_2x} \lambda(t)dt.$

(18) differentiating by x we get:

$$\nu(x) = \lambda_1(x)\nu(a_1 + b_1x) + \lambda_2(x)\nu(a_2 + b_2x) + f_1(x) \quad (19)$$

where

$$\nu(x) = (\rho(x) - c)\nu_2(x),$$

$$\lambda_1(x) = \frac{b_1\mu_1}{b_1\mu_1 + b_2\mu_2 - 1},$$

$$\lambda_2(x) = \frac{b_2\mu_2}{b_1\mu_1 + b_2\mu_2 - 1},$$

$$f_1(x) = \frac{1}{b_1\mu_1 + b_2\mu_2 - 1}\frac{d}{dx}f(x).$$

Relation (19) is a functional equation.

We will look for the solution of the functional equation (19) in the class of functions bounded at a point $x = 1$. If we abandon this requirement, then the corresponding homogeneous functional equation (19)

$$\nu(x) = \lambda_1(x)\nu(a_1 + b_1x) + \lambda_2(x)\nu(a_2 + b_2x) \quad (20)$$

may have a non-trivial solution.

Example. Let $p_1(x) = a + bx, p_2(x) = p_1(p_1(x)) = b^2x + ba + a$, where $a - b = c_1, c_1b + a = c_2$, then it is not difficult to make sure that the function

$$\nu(x) = (1 - x)^{\delta}, \text{ where } \delta = log_b \frac{\sqrt{\lambda_1^2 + 4\lambda_2} - \lambda_1}{2\lambda_2} \quad (21)$$

will be a nontrivial solution of the homogeneous equation (20), indeed, since

$$\nu(p_1(x)) = (1 - p_1(x))^{\delta} = b^{\delta}(1 - x)^{\delta},$$

$$\nu(p_2(x)) = (1 - p_2(x))^{\delta} = b^{2\delta}(1 - x)^{\delta},$$

then substituting these values into (20) we obtain the following quadratic equation$\lambda_2 b^{2\delta} + \lambda_1 b^{\delta} - 1 = 0.$

From here

$$\delta = log_b \frac{\sqrt{\lambda_1^2 + 4\lambda_2} - \lambda_1}{2\lambda_2}$$

and by virtue of condition (6) it is easy to make sure that $\delta < 0$, therefore, the solution (21) of the homogeneous functional equation (21) is not limited when $x = 1$.

Thus, the class of solutions where the solution of the functional equation (19) is sought is essential.

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