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TRANSVERSE VIBRATIONS OF A THREE-LAYER ELASTIC PLATE

***Abstract**: In the work, the heads of the displacements of the selected middle surface of the three-layer elastic plate are included as sought functions. Accordingly, a system of fifth-order differential equations has been developed that can be used to solve practical problems with respect to the functions sought by performing several mathematical operations.*
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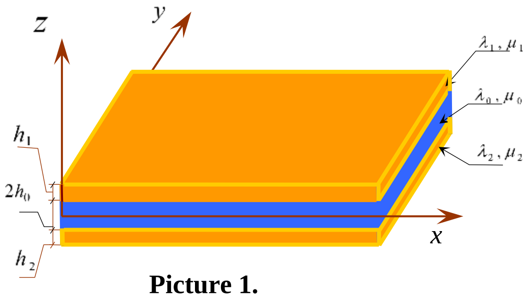
Introduction

Currently, several studies are being conducted simultaneously on multi-layer, especially three-layer plates. This is due to the fact that the three-layer plates maintain a high level of strength during various vibrations and easily solve economic problems. Such a collection of scientific papers can include many articles, including [1, 2].



Picture 1.

Multilayer plates, especially three-layer plates, are widely used in various fields of technology and construction. In most cases, the dynamic calculation of the plates is based on the classical theory based on Kirchhoff's hypotheses [3]. In some cases, the dynamic calculations are based on equations of the type S.P.Timoshenko, which take into account the transverse shear deformation and rotational inertia [4].

Over the next few decades, plate theories based on G.I Petrashen's exact solution method were developed. In particular, this method was developed by Professor IG Filippov [6] and his students in the theory of vibration of three-layer plates with an asymmetrical structure.

In this paper, the vibration equations of a three-layer elastic plate are given by the Petrashen-Filippov method mentioned above, but for a case where the problem is considered to be a flat problem. In addition to the vibration equations, an algorithm has been developed that allows determining the state of stress-strain in any section of the plate with a single value in coordinates and watts.

Problem statement. We look at a three-layer plate in the Cartesian coordinate system. The platelayers are made of different materials and the contact between them is considered a virgin. Assume that the plate is in a state of flat deformation at right angles Oxz (Figure 1). In this case, we direct the axis Ox along the line of contact of the layers of the cross section, and the axis Oz - perpendicular to it. We number the plate layers with “1”, “2” and “3” as in Figure 1. Let the thicknesses of the layers be h_0, h_1 and h_2, respectively, for the plate layer materials, the Lame coefficients $(\lambda_0,\mu_0),(\lambda_1,\mu_1)$ and (λ_2,μ_2), and the densities ρ_0, ρ_1 and ρ_2.

In the Cartesian coordinate system, we obtain the relations between the stresses and deformations at the points of the layers and the equations of motion for the points of the layers in the Cartesian coordinate system, i.e.

$$\sigma_{ii}^{(m)}=\lambda_m\left(\varepsilon^{(m)}\right)+2\mu_m\left(\varepsilon_{ii}^{(m)}\right),\quad \sigma_{ij}^{(m)}=\mu_m\left(\varepsilon_{ij}^{(m)}\right), \tag{1}$$

$$\sigma_{ij,j}^{m}+\rho_m\cdot F_i^{m}=\rho_m\cdot\frac{\partial^2 U_{mi}}{\partial t^2} \tag{2}$$

where $m=0,1,2$ is the layer number index;

Given that the potentials of the transverse and longitudinal waves [6] ψ_m and φ_m are the displacement vectors $\vec{U}^m=\vec{U}^m(U_m,W_m)$ of the points of the layers in the case of plane deformation, we introduce [7]:

$$\vec{U}^m=grad\,\varphi_m+rot\,\vec{\psi}_m \tag{3}$$

Here $\vec{i}$, $\vec{j}$, $\vec{k}$ are the unit axes of the coordinate axes. Putting these (3) expressions into (2) motion equations

$$\Delta\varphi_m=\frac{1}{a_m^2}\frac{\partial^2\varphi_m}{\partial t^2};\qquad \Delta\psi_m=\frac{1}{b_m^2}\frac{\partial^2\psi_m}{\partial t^2}, \tag{4}$$

we come to the wave equations.
Where

$$\Delta=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial z^2}$$

is the two-dimensional Laplace differential operator.

We assume that for $t<0$ times the plate was at rest, and at $t=0$ the dynamic loads began to act on its boundary surfaces. In this case, the boundary conditions are as follows:

When $z=\pm h_m$

$$\sigma_{xz}^{m}=f_x^{m}(x,t);\quad \sigma_{zz}^{m}=f_z^{m}(x,t);\quad \sigma_{yz}^{m}=0,\quad (m=0,1,2). \tag{5}$$

In addition, the following kinematic conditions are appropriate on the test surface:

$$U_0(x,z,t)\big|_{z=h_0}=U_1(x,z,t)\big|_{z=h_0};\quad W_0(x,z,t)\big|_{z=h_0}=W_1(x,z,t)\big|_{z=h_0}. \tag{6}$$

the initial conditions are assumed to be zero, i.e., when $t=0$

When $t=0$

$$\varphi_m=\psi_m=0;\qquad \frac{\partial\varphi_m}{\partial t}=\frac{\partial\psi_m}{\partial t}=0 \tag{7}$$

Thus, the solution of the problem of longitudinal oscillations of a three-layer plate leads to the integration of the system of equations (4) in the boundary conditions (5), (6) and in the initial conditions (7). To solve the problem, ψ_m and φ_m potential functions [5]

$$\varphi_m=\int_0^\infty\left\{\begin{matrix}\sin kx\\-\cos kx\end{matrix}\right\}dk\int_{(l)}\tilde{\varphi}_m e^{pt}dp;$$

$$\psi_m=\int_0^\infty\left\{\begin{matrix}\cos kx\\ \sin kx\end{matrix}\right\}dk\int_{(l)}\tilde{\psi}_m e^{pt}dp,\quad (m=0,1,2). \tag{8}$$

and put them in (4)

$$\frac{d^2\tilde{\varphi}_m}{dz^2}-\alpha_m^2\tilde{\varphi}_m=0;\quad \frac{d^2\tilde{\psi}_m}{dz^2}-\alpha_m^2\tilde{\psi}_m=0\quad (m=0,1,2) \tag{9}$$

we get the equations. Here

$$\alpha_m^2=k^2+\frac{1}{a_m^2}p^2;\quad \beta_m^2=k^2+\frac{1}{b_m^2}p^2 \tag{10}$$

Under the influence of the above (5) - symmetrical loads, the plate oscillates longitudinally, and the solutions of equations (9) consist of

$$\tilde{\varphi}_m(z,k,p)=A_m^{(1)}ch\alpha_m z,$$

$$\tilde{\psi}_m(z,k,p)=B_m^{(1)}sh\beta_m z. \quad (m=0,1,2) \qquad (11)$$

We also describe the displacements of the points of the layers in (8) and have

$$\tilde{U}_m=kA_m^{(1)}ch(\alpha_m z)-\beta_m B_m^{(1)}ch(\beta_m z);$$

$$\tilde{W}_m=\alpha_m A_m^{(1)}sh(\alpha_m z)-kB_m^{(1)}sh(\beta_m z). \quad (m=0,1,2) \qquad (12)$$

for the replaced $\tilde{U}_m$, $\tilde{W}_m$. We line the right-hand sides of these expressions (12) in degrees $(\alpha_m z)$ and $(\beta_m z)$

$$\tilde{W}_m=\sum_{n=0}^{\infty}\left[\alpha_m^{2n+2}A_m^{(1)}-k\beta_m^{2n+1}B_m^{(1)}\right]\frac{z^{2n+1}}{(2n+1)!};$$

$$\tilde{U}_m=\sum_{n=0}^{\infty}\left[k\alpha_m^{2n}A_m^{(1)}-\beta_m^{2n+1}B_m^{(1)}\right]\frac{z^{2n}}{(2n)!} \qquad (13)$$

As the search functions in the oscillation equations of the three-layer plate, we choose the main parts of the displacements and displacements, i.e.,

$$\tilde{U}_0^{(0)}=kA_0^{(1)}-\beta_0 B_0^{(1)},\ \tilde{W}_0^{(0)}=\left[\alpha_0^2 A_0^{(1)}-k\beta_0 B_0^{(1)}\right]\xi.$$

From here

$$A_0^{(1)}=\frac{\frac{1}{\xi}\tilde{W}_0^{(0)}-k\tilde{U}_0^{(0)}}{\alpha_0^2-k^2};$$

$$\beta_0 B_0^{(1)}=\frac{\frac{k}{\xi}\tilde{W}_0^{(0)}-\alpha_0^2\tilde{U}_0^{(0)}}{\alpha_0^2-k^2}; \qquad (14)$$

We obtain a system of equations by substituting the above expressions (12) for the alternating $\tilde{U}_m$ and $\tilde{W}_m$ transitions into the contact conditions (6). Solving this system of equations, we express the variables $A_1^{(1)}$ and $B_1^{(1)}$ by $A_0^{(1)}$ and $B_0^{(1)}$. Then we add (13) to the resulting expressions

$$A_1^{(1)}=\frac{1}{(\alpha_0^2-k^2)\Delta_1^0}\left[\frac{1}{\xi}\left(\Delta_{11}^0+\frac{k}{\beta_0}\Delta_{12}^0\right)\tilde{W}_0^{(0)}-\left(k\Delta_{11}^0+\frac{\alpha_0^2}{\beta_0}\Delta_{12}^0\right)\tilde{U}_0^{(0)}\right];$$

$$B_1^{(1)}=\frac{1}{(\alpha_0^2-k^2)\Delta_1^0}\left[\frac{1}{\xi}\left(\Delta_{21}^0+\frac{k}{\beta_0}\Delta_{22}^0\right)\tilde{W}_0^{(0)}-\left(k\Delta_{21}^0+\frac{\alpha_0^2}{\beta_0}\Delta_{22}^0\right)\tilde{U}_0^{(0)}\right]. \qquad (15)$$

To find non-zero voltages $\sigma_{xz}^{(m)}$, $\sigma_{zz}^{(m)}$ at any point of the plate layers, we describe them in the same way as (8). Then by substituting (8) for (1) on the other side and equating it with the expression described as (8) we get the following from the contact condition (5)

$$\tilde{M}_1\left(2k\alpha_1 A_1^{(1)}(k,p)sh(\alpha_1 z)-(\beta_1^2+k^2)B_1^{(1)}(k,p)sh(\beta_1 z)\right)=\tilde{f}_x^{(1)}(k,p);$$

$$\left[\tilde{L}_1(\alpha_1^2-k^2)+2\tilde{M}_1k^2\right]A_1^{(1)}(k,p)ch(\alpha_1 z)-2\tilde{M}_1k\beta_1B_1^{(1)}(k,p)ch(\beta_1 z)=\tilde{f}_z^{(1)}(k,p). \qquad (16)$$

By substituting the values of $A_1^{(1)}$ and $B_1^{(1)}$, defined by formulas (15), into the last (16) relationship, and by extending the hyperbolic functions in the resulting equations into power series along the thickness coordinate levels, we obtain the general equations of longitudinal oscillations of a three-layer plate. Since the order of these equations by derivatives is infinite, we assume that the conditions of the intersection of infinite-degree rows are satisfied in the work, and we limit ourselves to the first terms in the distributions. Then we have the following system of equations that can be used to solve practical problems.

$$-\left\{\left[A_{11}\frac{(h_0+h_1)h_0^4}{12}+A_{12}\frac{(h_0+h_1)^3h_0^2}{36}\right]\frac{\partial^4}{\partial t^4}-\left[A_{13}\frac{(h_0+h_1)h_0^4}{12}+A_{14}\frac{(h_0+h_1)^3h_0^2}{36}\right]\frac{\partial^4}{\partial x^2\partial t^2}+\right.$$

$$+\left[A_{15}\frac{(h_0+h_1)h_0^4}{12}+A_{16}\frac{(h_0+h_1)^3h_0^2}{36}\right]\frac{\partial^4}{\partial x^4}+\left[A_{17}\frac{(h_0+h_1)h_0^2}{6}+A_{18}\frac{(h_0+h_1)^3}{6}\right]\frac{\partial^2}{\partial t^2}-$$

$$-\left[A_{19}\frac{(h_0+h_1)h_0^2}{6}+A_{110}\frac{(h_0+h_1)^3}{6}\right]\times\frac{\partial^2}{\partial x^2}+A_{111}(h_0+h_1)\Bigg\}\frac{1}{\xi}\frac{\partial}{\partial x}W_0^{(0)}-$$

$$=\left\{\left[B_{11}\frac{(h_0+h_1)h_0^2}{6}+B_{12}\frac{(h_0+h_1)^3}{6}\right]\frac{\partial^4}{\partial t^4}-\left[B_{13}\frac{(h_0+h_1)h_0^2}{6}+B_{14}\frac{(h_0+h_1)^3}{6}\right]\frac{\partial^4}{\partial x^2\partial t^2}+\right.$$

$$\left.+\left[B_{15}\frac{(h_0+h_1)h_0^2}{6}-B_{16}\frac{z^3}{6}\right]\frac{\partial^4}{\partial x^4}+B_{17}(h_0+h_1)\frac{\partial^2}{\partial t^2}-B_{18}(h_0+h_1)\frac{\partial^2}{\partial x^2}\right\}U_0^{(0)}=$$

$$=\left\{S_1\frac{h_0^4}{12}\frac{\partial^4}{\partial t^4}-S_2\frac{h_0^4}{12}\frac{\partial^4}{\partial x^2\partial t^2}+\frac{h_0^4}{12}\frac{\partial^4}{\partial x^4}+S_3\frac{h_0^2}{6}\frac{\partial^2}{\partial t^2}-S_4\frac{h_0^2}{6}\frac{\partial^2}{\partial x^2}+1\right\}f_x^{(1)}(k,p);\quad(17)$$

$$\left\{\left[A_{21}\frac{h_0^4}{12}+A_{22}\frac{h_0^2(h_0+h_1)^2}{12}\right]\frac{\partial^4}{\partial t^4}-\left[A_{23}\frac{h_0^4}{12}-A_{24}\frac{h_0^2(h_0+h_1)^2}{12}\right]\frac{\partial^4}{\partial x^2\partial t^2}+\right.$$

$$+\left[A_{25}\frac{h_0^4}{12}+A_{26}\frac{h_0^2(h_0+h_1)^2}{12}\right]\frac{\partial^4}{\partial x^4}+\left[A_{27}\frac{h_0^2}{6}+A_{28}\frac{(h_0+h_1)^2}{2}\right]\frac{\partial^2}{\partial t^2}-$$

$$\left.-\left[A_{29}\frac{h_0^2}{6}+A_{210}\frac{(h_0+h_1)^2}{2}\right]\frac{\partial^2}{\partial x^2}+A_{211}\right\}\frac{1}{\xi}W_0^{(0)}+\left\{\left[B_{21}\frac{h_0^4}{12}+B_{22}\frac{h_0^2(h_0+h_1)^2}{12}\right]\frac{\partial^4}{\partial t^4}-\right.$$

$$-\left[B_{23}\frac{h_0^4}{12}+B_{24}\frac{h_0^2(h_0+h_1)^2}{12}\right]\frac{\partial^4}{\partial x^2\partial t^2}+\left[B_{25}\frac{h_0^4}{12}+B_{26}\frac{h_0^2(h_0+h_1)^2}{12}\right]\frac{\partial^4}{\partial x^4}+$$

$$\left.+\left[B_{27}\frac{h_0^2}{6}+B_{28}\frac{(h_0+h_1)^2}{2}\right]\frac{\partial^2}{\partial t^2}-\left[B_{29}\frac{h_0^2}{6}+B_{210}\frac{(h_0+h_1)^2}{2}\right]\frac{\partial^2}{\partial x^2}-B_{211}\right\}\frac{\partial}{\partial x}U_0^{(0)}=$$

$$=\left\{S_1\frac{h_0^4}{12}\frac{\partial^4}{\partial t^4}-S_2\frac{h_0^4}{12}\frac{\partial^4}{\partial x^2\partial t^2}+\frac{h_0^4}{12}\frac{\partial^4}{\partial x^4}+S_3\frac{h_0^2}{6}\frac{\partial^2}{\partial t^2}-S_4\frac{h_0^2}{6}\frac{\partial^2}{\partial x^2}+1\right\}f_z^{(1)}(k,p).$$

Here, the coefficients A_{ij}, B_{ij}, S_{ij} $(i,j=1,2)$ are variables that depend on the elastic properties of the layers. By solving this system of equations, it is possible to find the functions sought and to find the displacements and stresses that occur in the layers of a three-layer plate.

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