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SIMULATION OF THE PROPAGATION OF ACTIVE AEROSOL PARTICLES IN THE ATMOSPHERE METHOD OF CHANGE OF VARIABLES FOR ITS NUMERICAL INTEGRATION

***Abstract**: The article discusses as a result, a multidimensional system of partial differential equations was obtained that describes the process of propagation of active aerosol particles in the atmosphere, which takes into account the main factors, that is, the rates of the forward and reverse reactions, under the action of chemical reactions, as well as the absorption of aerosol particles in the atmosphere and other weather conditions. climatic factors, using which it is possible to calculate and predict the concentration of harmful substances over time.*

***Key words**: rate of deposition of particles, the process of transfer and diffusion of aerosol particles in the atmosphere, numerical algorithm.*

***Language**: English*

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Introduction

The process of movement and settling of particles in turbulent atmospheric flows over heat-producing sources was theoretically studied by Smirnov N.N. and his colleagues [1]. The mathematical model developed by the authors takes into account the effects of two-way interaction of the "gas-particle" system and combines deterministic and stochastic approaches. A modified turbulent flow model is used to describe the behavior of the gas phase. The system of equations describing the turbulent flow was obtained by Favre averaging and includes the mass balance in the gas phase, the mass balance of the k th component, and the balance of momentum and energy. The equations of motion of particles, in addition to the forces of gravity, resistance and the Archimedes force, take into account random turbulent pulsations in the gas flow, the characteristics of which are determined using the solution obtained in the framework of the model k - ε. The obtained results of numerical experiments made it possible to determine the effect of heat release sources on the dispersion and character of particle deposition.

In [2] an attempt was made to study atmospheric dispersion taking into account the sedimentation rate of particles, when the shape of the particles serves as an input parameter of the model. The authors used a semi-empirical formula for non-spherical particles to determine the sensitivity of the volcanic ash cloud transport process to particle shape. The process was modeled using the Lagrangian atmospheric dispersion model. It has been established that there is no noticeable difference in the vertical trajectories of spherical and non-spherical particles 1 μm in size. The vertical motion of 10 μm particles is more sensitive to shape, but the similar motion pattern of spherical and non-spherical particles is preserved, with the settling velocity always positive, and particles moving both downwards and upwards, indicating the predominance of advection and turbulent diffusion. Shape sensitivity increases dramatically with size, such that non-spherical 100 μm particles settle much more slowly and can move along the plume axis 44% farther from the source than spherical particles. The proposed approach makes it possible to more accurately predict the concentration of fly ash in the atmosphere and the distance of its movement.

Naslund E. and Thaning L. presented a solution to the problem of determining the rate of particle settling under unstable atmospheric stratification [3]. The equation of motion for solid spherical particles uses a common drag coefficient. Time constants, stopping times, and settling rates for stable atmospheric stratification are calculated for a wide range of Reynolds numbers. The resulting settling time is compared with the time calculated for the case where settling occurs by atmospheric convection. It has been found that in this case the solid spherical particles will take much longer to settle, resulting in an increase in the drag coefficient caused by an increase in the relative velocity between the particles and the air mass flow. Such amplification is present both for the horizontal wind field, due to the relationship between the movement of particles in different directions, and for the vertical field. The effect is most pronounced in the region of intermediate Reynolds numbers, slightly above the Stokes range, where the increase in settling time can be more than 10% for certain frequencies and amplitudes of turbulent fluctuations.

It should be noted that in problems where physical processes can be identified in an obvious way, for example, the transfer of a substance in the direction of the wind and molecular diffusion, then in this case it would be quite reasonable to use the method of splitting by physical processes at each time layer.

The idea of the splitting method is to reduce the original multidimensional problem to problems of a simpler structure, which are then sequentially or in parallel solved by known numerical methods. This can be achieved in a variety of ways, so by now a large number of different additive difference schemes have been created [4].

For example, in [5] presents an exact numerical method for a class of scalar strongly degenerate convection-diffusion equations. The method is based on the splitting of the convective and diffusion terms. The non-linear convective part is solved using front tracking and size splitting, while the non-linear diffusion part is solved by a semi- implicit finite difference scheme. The method proposed by the authors has a built-in mechanism for detecting and correcting non-physical loss of entropy, which can occur if the time step is large. The authors demonstrate in this paper that the splitting method accurately resolves sharp gradients, can use large time steps, and has first-order convergence. Although the method has small mass balance errors, it is nevertheless quite efficient.

The task was implemented according to the central difference scheme of A. Kurganov and E. Tadmor for accurate modeling of advection reaction processes. The main partial differential equation is divided into two subproblems, which are solved sequentially during each time step. Unlike traditional methods, the scheme proposed by the authors provides a very efficient method for solving the advection-diffusion-reaction equation for any value of the Peclet number. The performed analysis of mass balance errors shows that the unconventional scheme proposed by the authors has a splitting error, which differs from the error that occurs in traditional schemes. Numerical results demonstrate sufficient reliability of the proposed scheme.

Geiser J. and Kravvaritis Ch . presented a new approach to constructing a domain decomposition method based on the iterative splitting method [6]. The proposed two-stage iterative splitting scheme for solving a linear evolutionary equation is based on separation in space and time. The authors studied the convergence properties of the method. The effectiveness of the proposed method is shown by comparing the numerical results with the additive Schwartz wave relaxation method.

The application of the physical splitting method for modeling the process of distribution of harmful substances in the atmosphere was studied in the works of Havasi A. and Farago I. [7]. To illustrate the physical splitting method, the authors use the Danish Euler model. The basic equation for the transport of pollutants over long distances

$$\frac{\partial c}{\partial t} + \nabla\left(\underline{u}c\right) = \nabla\left(\underline{K}\nabla c\right) + R\left(\underline{x},c\right) + E + gc$$

with appropriate initial and boundary conditions is reduced to several subsystems describing physical processes: horizontal transport, horizontal diffusion, chemical reactions with ejection, sedimentation and vertical exchange. The authors pay special attention to the issue of errors arising as a result of the separation procedure. The authors give several cases when this error disappears, but emphasize that in practice this rarely happens, so it is important to analyze and carefully select the schemes used so that the error is as small as possible.

In the article [8] a mathematical model and a numerical calculation algorithm for analyzing and predicting the process of gas filtration in a porous medium by the coordinate splitting method are presented, and an estimate of its effectiveness is given in comparison with the calculation algorithm by the alternating directions method.

An analysis of the reviewed literary sources shows that by now many authors have already obtained significant scientific results and have developed various approaches and methods for modeling the process under study. However, the analysis also shows that there are certain gaps in this area. In particular, mathematical models of the processes of transfer and diffusion of aerosol emissions in the atmosphere in the multidimensional setting, taking into account changes in the deposition rates of fine particles and weather and climatic factors, as well as the development of efficient conservative numerical algorithms based on methods with a high order of approximation in time and space variables.

The impact on human health caused by the spread of primary and secondary pollutants in the atmospheric air is mainly associated with the respiratory and cardiovascular systems, but can also have far-reaching consequences in the form of cancer and genetic changes [9]. According to research results published in the reports of the US National Academy of Sciences, air pollution leads to premature deaths of up to 8.8 million people every year [10]. The highest mortality rate in this regard is observed in India and China, according to some estimates, up to 500 thousand people annually [11].

Transport systems (including road, rail, sea, river and air), especially in cities, are the largest sources of air pollution. According to many authors, the share of all types of transport can reach up to 60% of the total volume of anthropogenic emissions [12]. Moreover, atmospheric emissions from road transport are an order of magnitude higher than emissions from all other types of vehicles. Given the trend of steady growth in the car fleet, the environmental situation is aggravated in most countries. Gaseous mixtures and solid aerosol particles emitted as a result of the operation of internal combustion engines contain more than 200 types of pollutants, including carcinogens.

Industrial production is the next major group of anthropogenic emission sources. The share of all types of industry on average accounts for up to 40% of the total volume of artificial emissions of pollutants into the atmosphere. This group of sources is extremely diverse both in terms of the composition of industries and the list of emitted pollutants. The largest contribution to air pollution here is made by enterprises of the fuel and energy sector, objects of the metallurgical and chemical industries, mining and construction industries [13].

Household services, including various enterprises of housing and communal services, are sources of pollution in the urban environment. The main sources of air pollution in this group include boiler houses of central heating systems. Depending on the type of fuel used (gas, fuel oil, firewood, coal, etc.), these sources emit into the atmosphere: sulfur dioxide, nitrogen oxides, heavy metals, volatile organic compounds (VOCs), organic hydrocarbons, as well as various in chemical composition solid large and fine particles [13].

Finally, it is worth noting that over the past few decades, the agricultural industry has become on a par with, and in some cases even surpasses, other sources in terms of emissions into the atmosphere. This situation is especially typical for countries with a powerfully developed agricultural sector, for example, the USA, Russia, China, European countries. According to research by scientists at Columbia University (USA), nitrogen-enriched fertilizers and animal waste are among the largest sources of this group of emissions. The combination of such wastes with combustion products forms fine particles, which are very dangerous for the health and life of the population [14].

Sources of emissions into the atmosphere related to household and agricultural groups, depending on weather and climatic conditions, topography characteristics or development of the area, often form a locally limited unfavorable environmental situation, but can also make a certain contribution to the process of transboundary transport of pollutants in the atmosphere.

In table. Clause 2.5 of Annex 2 provides quantitative indicators of global emissions of major pollutants into the atmosphere by groups of anthropogenic sources (data for 2000) [15]. To date, the situation shown in Table. Clause 2.5 varies somewhat for one or another group, due to the efforts associated with the introduction of environmentally friendly technologies in various sectors of activity. However, unfortunately, the general trend towards an increase in emissions persists.

The data given in table. Clauses 2.3-P.2.5 of Appendix 2 correspond proportionally to the existing situation in Uzbekistan. As in other developed countries, the industry, transport complex and agricultural sector of Uzbekistan play a crucial role in the development of the economy and improving the welfare of the population of our republic. However, enterprises in the mining, metallurgical, petrochemical, chemical and other industries often exceed the current standards for the emission of harmful substances into the atmosphere, and sometimes not several times, but dozens of times.

According to the State Committee for Nature Protection Uzbekistan [16], the level of atmospheric air pollution on the territory of the Republic of Uzbekistan is determined by emissions from stationary and mobile sources. The volume of emissions of pollution into the atmosphere as a whole in Uzbekistan in 2009 amounted to 2125.0 thousand tons, of which: stationary sources accounted for 741.0 thousand tons, or 34.8%, and mobile sources -1384.0 thousand tons, or 65.2% of the total volume of pollutants emitted into the atmospheric air.

As for natural sources of emissions into the atmosphere, in Uzbekistan they mainly include weakly fixed sandy soils of the Karakum and Kyzyl Kum deserts , as well as the drained part of the Aral Sea due to wind erosion.

In addition to the typing shown, sources of air pollution can also be divided into groups and classes depending on the characteristics of the pollutants themselves. There are usually three main groups:

- mechanical pollutants - solid dust particles of cement plants, rock dumps subject to soil erosion, soot from the combustion of boiler fuel, washable tires, etc.;
- chemical pollutants - gaseous substances and fine aerosols that can enter into chemical reactions under the influence of humidity and temperature;
- radioactive pollutants - radiation, isotopes, particles of radioactive dust.

According to the degree of danger to human health and the negative impact on the environment as a whole, pollutants are divided into several classes. According to the current standards in Uzbekistan, the following classification has been adopted [17]:

- Class I - especially dangerous substances (mercury, benz (o) pyrene , hexachlorane, chromium oxides, etc.);
- Class II - hazardous substances (chlorine, calcium fluoride, hydrogen sulfide, formaldehyde, etc.);
- Class III - low-hazard substances (dust, sulfur dioxide, zinc, acetaldehyde, etc.);
- Class IV - non-hazardous substances (ammonia, carbon oxides, dimethyl sulfide, etc.).

The hygienic standard SanPiN RUz N 0293-11 stipulates the values of maximum permissible concentrations (MPC) of harmful substances in the atmospheric air for 485 substances. At the same time, hazard class I substances make up 43 (8.94%); II class - 157 (32.64%), III class - 192 (39.91%) and IV class - 89 (18.5%). Of these, for 199 substances, taking into account the indicator of harmfulness, only the maximum one-time maximum is established, and for 282 substances - the maximum one-time, average daily, average monthly, average annual MPC.

In addition, this hygienic standard contains a list of 39 substances whose release into the atmospheric air is prohibited due to the extremely high biological activity of these substances. Also given are data on summation groups - substances that have a summation effect when they are present in the atmospheric air together.

In conclusion of the paragraph, we will consider the characteristics of some of the main pollutants taken into account when assessing the quality of atmospheric air.

CO_2 - carbon dioxide or carbon dioxide - is formed as a result of the combustion of fossil fuels and biomass, such as coal, oil, natural gas, synthetic fuels, wood. CO_2 - this is the main component of the number of triatomic gases that affect the development of the greenhouse effect.

As a result of incomplete combustion of fuel, carbon monoxide (CO) is also released, a toxic gas that adversely affects the human cardiovascular system. When inhaled , due to the double bond present in its molecule, strong compounds are formed with human blood hemoglobin and, thereby, the flow of oxygen into the blood is blocked. Carbon monoxide belongs to the 4th hazard class and has a one-time maximum allowable concentration of 5.0 mg/ m^3 .

SO_2 - Sulfur dioxide or sulfur dioxide is one of the most toxic gaseous substances, accounting for more than 90% of the emissions of sulfur compounds with flue gases. The largest amount of sulfur contains coal and heavy types of petroleum products. Sulfur dioxide affects oxidation, destroys materials, adversely affects human health. Long-term exposure to sulfur dioxide on a person leads to a loss of taste sensations, shortness of breath, and then to pneumonia, interruptions in cardiac activity, circulatory disorders and respiratory arrest. The duration of this substance in the atmosphere is relatively short (up to 15-20 days), however, given the presence of oxygen and moisture, SO_2 it is often converted into sulfuric acid, which then falls out with precipitation in the form of acid rain. Sulfur dioxide belongs to the III class of danger, the norm of one-time MPC is 0.5 mg/ m^3 .

NO_x - nitrogen oxides - are formed during the combustion of any type of fossil fuel containing nitrogen compounds, as well as as a result of the oxidation of nitrogen in the air during the combustion process. The higher the combustion temperature, the more intense the formation of nitrogen oxides. Also, enterprises producing nitrogen fertilizers, nitric acid and nitrates, aniline dyes, nitro compounds can also be a source of nitrogen oxides. Nitrogen oxides adversely affect human health, play a role in the formation of the greenhouse effect and the destruction of the ozone layer. In addition, nitrogen oxides are the cause of forest extinction, acid rain and other negative environmental effects. Nitrogen dioxide NO_2 belongs to hazard class II, has a one-time maximum concentration limit equal to 0.085 mg/ m^3.

O_3 Ozone is a gas that has a characteristic odor (after rain). It is a stronger oxidizing agent than oxygen. Ozone is considered the most toxic of all common air pollutants. In the lower atmospheric layer, ozone is formed as a result of photochemical processes involving nitrogen dioxide and volatile organic compounds. Ozone is involved in the formation of the greenhouse effect, negatively affects human health, crops, etc. According to SanPiN standards, ozone belongs to hazard class I, having a one-time MPC of 0.16 mg/m^3 .

Freons are gaseous impurities with a small proportion in the atmosphere. They are released into the atmosphere mainly during the production of thermal insulation materials, foam plastic, etc., they are emitted from the refrigerants of refrigerators, air conditioners, etc. Freons are destroyers of the ozone layer of the atmosphere. They increase the level of ultraviolet radiation of the Earth and contribute to the formation of the greenhouse effect.

Particles of heavy metals such as chromium, manganese, iron, cobalt, nickel, copper, zinc, cadmium, tin, mercury, lead, etc. have been studied. The term "heavy metals" in this context is considered from a medical and environmental point of view, taking into account biological activity and toxicity of these elements. Many heavy metals (iron, copper, zinc, etc.) are involved in biological processes and, in certain amounts, are essential microelements for the life of plants, animals, and humans. However, due to the fact that these elements have the ability to accumulate in the final environment, for example, in body tissues, they can subsequently have a harmful effect on human health, causing a number of diseases. Metals such as lead and mercury are generally defined as toxic because they have no useful role in biological processes.

Solid particles of the type PM_{10} and $PM_{2,5}$ enter the atmosphere in the form of combustion products (smoke, soot, soot) or through removal from the earth's surface. The degree of air pollution with mechanical impurities depends on the type of ground cover (sands, alumina, concrete, asphalt), its sanitary condition (cleanliness, humidity), as well as on the quality of filtration and purification of industrial emissions. Solid dust particles cause irritation of the mucous membranes of the upper respiratory tract and eyes. Therefore, their prolonged exposure causes respiratory diseases. Suspended fine dust is the most hazardous to health, as it is able to penetrate into the lungs and linger in them for a long time. Particles of heavy metals and other toxic substances contained in the dust inevitably lead to poisoning of varying severity.

Whether dust belongs to one or another MPC class and the accepted MPC standards depend on its characteristics. For example, the dust of the Aral salts is included in class III of pollutants at a single MPC rate of 0.5 mg/ m^3 , and dust from lead-zinc production with a lead content of up to 1% belongs to class I and has a single MPC rate of 0.001 mg /m^3. cubic meters.

2. Statement of the problem

Since the process of transfer and diffusion of harmful substances in the atmosphere, taking into account the weather and climatic factors acting on it, the orography of the terrain and other disturbances, is a complex non-stationary process, a more adequate mathematical model was developed for its study. In addition to taking into account essential parameters, this model is considered in a three-dimensional setting and is described on the basis of the basic laws of hydromechanics using a three- dimensional partial differential equation [18, 19, 20, 21, 22, 23, 24, 25, 26, 27]:

$$\frac{\partial\theta}{\partial t}+u\frac{\partial\theta}{\partial x}+v\frac{\partial\theta}{\partial y}+\left(w-w_g\right)\frac{\partial\theta}{\partial z}+\sigma\theta=\mu\left(\frac{\partial^2\theta}{\partial x^2}+\frac{\partial^2\theta}{\partial y^2}\right)+\frac{\partial}{\partial z}\left(\kappa\frac{\partial\theta}{\partial z}\right)+\delta Q; \quad (1)$$

$$\frac{dw_g}{dt}=\frac{mg-6\pi\gamma rw_g-0,5c\rho sw_g^2}{m} \quad (2)$$

with the corresponding initial and boundary conditions:

$$\theta\big|_{t=0}=\theta^0; \quad w_g\big|_{t=0}=w_g^0 \; ; \quad (3)$$

$$-\mu\frac{\partial\theta}{\partial x}\bigg|_{x=0}=\xi\left(\theta_E-\theta\right); \qquad \mu\frac{\partial\theta}{\partial x}\bigg|_{x=Lx}=\xi\left(\theta_E-\theta\right); \quad (4)$$

$$-\mu\frac{\partial\theta}{\partial y}\bigg|_{y=0}=\xi\left(\theta_E-\theta\right); \qquad \mu\frac{\partial\theta}{\partial y}\bigg|_{y=L_y}=\xi\left(\theta_E-\theta\right); \quad (5)$$

$$-\kappa \frac{\partial \theta}{\partial z}\bigg|_{z=0} = \left(\beta\theta - f_0\right); \quad \kappa \frac{\partial \theta}{\partial z}\bigg|_{z=H_z} = \xi\left(\theta_E - \theta\right). \tag{6}$$

Here x, y, z is the coordinate system; u, v, w - wind speed in three directions [28, 29].

For the numerical solution of the problem (1)-(6), we assume that the u, v, w time functions are piecewise homogeneous.

Because Since the task is described by a system of nonlinear partial differential equations, it is difficult to obtain an analytical solution. Considering the above, when solving the problem numerically, in order to increase the order of approximation in spatial variables, we introduce the following notation [18, 19, 30, 31, 32, 24, 33]:

$$\bar{w} = w - w_g\,; \theta = e^{\frac{ux+vy}{2\mu} + \frac{\bar{w}z}{2\kappa}} \tilde{\theta}, \tag{7}$$

and instead of equation (1) we get

$$\frac{\partial \tilde{\theta}}{\partial t} + \sigma_1 \tilde{\theta} = \mu \frac{\partial^2 \tilde{\theta}}{\partial x^2} + \mu \frac{\partial^2 \tilde{\theta}}{\partial y^2} + \frac{\partial}{\partial z}\left(\kappa \frac{\partial \tilde{\theta}}{\partial z}\right) + e_1 \delta Q, \tag{8}$$

where

$$\sigma_1 = \frac{\kappa u^2 + \kappa v^2 + \mu \bar{w}^2 + 4\sigma\mu\kappa}{4\mu\kappa}; \quad e_1 = e^{-\left(\frac{ux+vy}{2\mu} + \frac{\bar{w}z}{2\kappa}\right)}.$$

Taking into account (8), respectively, the initial and boundary conditions have the following form:

$$\tilde{\theta}\Big|_{t=0} = \tilde{\theta}^0; \quad \mathrm{w_g}\Big|_{t=0} = w_g^0; \tag{9}$$

$$-\mu \frac{\partial \tilde{\theta}}{\partial x}\bigg|_{x=0} = \xi\left(e_1\theta_E - \tilde{\theta}\right); \qquad \mu \frac{\partial \tilde{\theta}}{\partial x}\bigg|_{x=L_x} = \xi\left(e_1\theta_E - \tilde{\theta}\right); \tag{10}$$

$$-\mu \frac{\partial \tilde{\theta}}{\partial y}\bigg|_{y=0} = \xi\left(e_1\theta_E - \tilde{\theta}\right); \qquad \mu \frac{\partial \tilde{\theta}}{\partial y}\bigg|_{y=L_y} = \xi\left(e_1\theta_E - \tilde{\theta}\right); \tag{11}$$

$$-\kappa \frac{\partial \tilde{\theta}}{\partial z}\bigg|_{z=0} = \left(\beta\tilde{\theta} - e_1 f_0\right); \qquad \kappa \frac{\partial \tilde{\theta}}{\partial z}\bigg|_{z=H_z} = \xi\left(e_1\theta_E - \tilde{\theta}\right). \tag{12}$$

3. Methods for solving the problem

Numerical solution of problem (8)-(12), the area of change of the desired variable (concentration of harmful substances), taking into account the boundary conditions, will be covered with a square grid area with a step $\Delta x; \ \Delta y; \Delta z$:

$$\Omega_{xyzt} = \Big\{\left(x_i = i\Delta x,\ y_j = j\Delta y,\ z_k = k\Delta z,\ \tau_n = n\ \Delta t\right);$$
$$i = \overline{1, N_x};\ \ j = \overline{1, M_y},\ \ k = \overline{1, L_z},\ n = \overline{0, N_t},\ \ \Delta t = \frac{1}{N_t}\Big\}.$$

To solve the problem (8)-(12), we divide equation (8) into three directions in time Δt and approximate:

direction $\_O\ x$:

$$\frac{\tilde{\theta}^{n+1/3}_{i,j,k} - \tilde{\theta}^{n}_{i,j,k}}{\Delta t/3} + \sigma_1 \tilde{\theta}^{n+1/3}_{i,j,k} = \frac{\mu}{\Delta x^2}\left(\tilde{\theta}^{n+1/3}_{i+1,j,k} - 2\tilde{\theta}^{n+1/3}_{i,j,k} + \tilde{\theta}^{n+1/3}_{i-1,j,k}\right) +$$
$$+ \frac{\mu}{\Delta y^2}\left(\tilde{\theta}^{n}_{i,j+1,k} - 2\tilde{\theta}^{n}_{i,j,k} + \tilde{\theta}^{n}_{i,j-1,k}\right) +$$
$$+ \frac{1}{\Delta z^2}\left(\kappa_{k+0,5}\tilde{\theta}^{n}_{i,j,k+1} - \left(\kappa_{k-0,5} + \kappa_{k+0,5}\right)\tilde{\theta}^{n}_{i,j,k} + \kappa_{k-0,5}\tilde{\theta}^{n}_{i,j,k-1}\right) + \frac{1}{3} e_1 \delta_{i,j,k} Q.$$

We tear off the brackets and, grouping similar terms of the equation, we obtain

$$\frac{\mu}{\Delta x^2}\tilde{\theta}^{n+1/3}_{i-1,j,k} - \left(\frac{3}{\Delta t} + \sigma_1 + \frac{2\mu}{\Delta x^2}\right)\tilde{\theta}^{n+1/3}_{i,j,k} + \frac{\mu}{\Delta x^2}\tilde{\theta}^{n+1/3}_{i+1,j,k} =$$
$$= -\left(\left(\frac{3}{\Delta t} - \frac{2\mu}{\Delta y^2} - \frac{\kappa_{k-0,5}+\kappa_{k+0,5}}{\Delta z^2}\right)\tilde{\theta}^{n}_{i,j,k} + \frac{\mu}{\Delta y^2}\tilde{\theta}^{n}_{i,j-1,k} + \frac{\mu}{\Delta y^2}\tilde{\theta}^{n}_{i,j+1,k} + \right.$$
$$\left. + \frac{\kappa_{k-0,5}}{\Delta z^2}\tilde{\theta}^{n}_{i,j,k-1} + \frac{\kappa_{k+0,5}}{\Delta z^2}\tilde{\theta}^{n}_{i,j,k+1} + \frac{1}{3}e_1\delta_{i,j,k}Q\right).$$

To simplify the above equation, we introduce the following notation:

$$a_{i,j,k} = \frac{\mu}{\Delta x^2};\quad b_{i,j,k} = \frac{3}{\Delta t} + \sigma_1 + \frac{2\mu}{\Delta x^2};\quad c_{i,j,k} = \frac{\mu}{\Delta x^2};$$
$$d_{i,j,k} = \left(\frac{3}{\Delta t} - \frac{2\mu}{\Delta y^2} - \frac{\kappa_{k-0,5}+\kappa_{k+0,5}}{\Delta z^2}\right)\tilde{\theta}^{n}_{i,j,k} + \frac{\mu}{\Delta y^2}\tilde{\theta}^{n}_{i,j-1,k} + \frac{\mu}{\Delta y^2}\tilde{\theta}^{n}_{i,j+1,k} +$$
$$+ \frac{\kappa_{k-0,5}}{\Delta z^2}\tilde{\theta}^{n}_{i,j,k-1} + \frac{\kappa_{k+0,5}}{\Delta z^2}\tilde{\theta}^{n}_{i,j,k+1} + \frac{1}{3}e_1\delta_{i,j,k}Q.$$

As a result, we obtain a tridiagonal system of linear algebraic equations:

$$a_{i,j,k}\tilde{\theta}^{n+1/3}_{i-1,j,k} - b_{i,j,k}\tilde{\theta}^{n+1/3}_{i,j,k} + c_{i,j,k}\tilde{\theta}^{n+1/3}_{i+1,j,k} = -d_{i,j,k}.$$

In the boundary condition (10) we use a second-order approximation and have

$$-\mu\frac{-3\tilde{\theta}^{n+1/3}_{0,j,k} + 4\tilde{\theta}^{n+1/3}_{1,j,k} - \tilde{\theta}^{n+1/3}_{2,j,k}}{2\Delta x} = \xi e_1\theta_E - \xi\tilde{\theta}^{n+1/3}_{0,j,k}$$

or

$$3\mu\tilde{\theta}^{n+1/3}_{0,j,k} - 4\mu\tilde{\theta}^{n+1/3}_{1,j,k} + \mu\tilde{\theta}^{n+1/3}_{2,j,k} = 2\Delta x e_1\xi\theta_E - 2\Delta\,x\,\xi\tilde{\theta}^{n+1/3}_{0,j,k}. \quad (13)$$

From the resulting tridiagonal system of linear algebraic equations

$$a_{1,j,k}\tilde{\theta}^{n+1/3}_{0,j,k} - b_{1,j,k}\tilde{\theta}^{n+1/3}_{1,j,k} + c_{1,j,k}\tilde{\theta}^{n+1/3}_{2,j,k} = -d_{1,j,k}$$

find $\tilde{\theta}^{n+1/3}_{2,j,k}$ as follows:

$$\tilde{\theta}^{n+1/3}_{2,j,k} = -\frac{a_{1,j,k}}{c_{1,j,k}}\tilde{\theta}^{n+1/3}_{0,j,k} + \frac{b_{1,j,k}}{c_{1,j,k}}\tilde{\theta}^{n+1/3}_{1,j,k} - \frac{d_{1,j,k}}{c_{1,j,k}}. \quad (14)$$

Equation (14) is substituted $\tilde{\theta}^{n+1/3}_{2,j,k}$ in (13) and found $\tilde{\theta}^{n+1/3}_{0,j,k}$ in the following form:

$$3\mu\tilde{\theta}^{n+1/3}_{0,j,k} - 4\mu\tilde{\theta}^{n+1/3}_{1,j,k} - \frac{a_{1,j,k}}{c_{1,j,k}}\mu\tilde{\theta}^{n+1/3}_{0,j,k} + \frac{b_{1,j,k}}{c_{1,j,k}}\mu\tilde{\theta}^{n+1/3}_{1,j,k} - \frac{d_{1,j,k}}{c_{1,j,k}}\mu =$$
$$= 2\Delta x e_1\xi\theta_E - 2\Delta\,x\,\xi\tilde{\theta}^{n+1/3}_{0,j,k};$$
$$\tilde{\theta}^{n+1/3}_{0,j,k} = \frac{4\mu c_{1,j,k} - b_{1,j,k}\mu}{3\mu c_{1,j,k} - a_{1,j,k}\mu + 2\Delta x\xi}\tilde{\theta}^{n+1/3}_{1,j,k} + \frac{d_{1,j,k} + 2\Delta x\xi c_{1,j,k}e_1\theta_E}{3\mu c_{1,j,k} - a_{1,j,k}\mu + 2\Delta x\xi}. \quad (15)$$

Using the above formulas (15), we find the values of the sweep coefficients $\alpha_{0,j,k}$ and $\beta_{0,j,k}$:

$$\alpha_{0,j,k} = \frac{4\mu c_{1,j,k} - b_{1,j,k}\mu}{3\mu c_{1,j,k} - a_{1,j,k}\mu + 2\Delta x\xi};\quad \beta_{0,j,k} = \frac{d_{1,j,k} + 2\Delta x\xi c_{1,j,k}e_1\theta_E}{3\mu c_{1,j,k} - a_{1,j,k}\mu + 2\Delta x\xi}. \quad (16)$$

Using the above operations in the boundary condition (10), we obtain

$$\mu\frac{\tilde{\theta}^{n+1/3}_{N-2,j,k} - 4\tilde{\theta}^{n+1/3}_{N-1,j,k} + 3\tilde{\theta}^{n+1/3}_{N,j,k}}{2\Delta x} = \xi e_1\theta_E - \xi\tilde{\theta}^{n+1/3}_{N,j,k}$$

or

$$\mu\tilde{\theta}^{n+1/3}_{N-2,\,j,k} - 4\mu\tilde{\theta}^{n+1/3}_{N-1,\,j,k} + 3\mu\tilde{\theta}^{n+1/3}_{N,\,j,k} = 2\Delta x e_1 \xi \theta_E - 2\Delta x\, \xi \tilde{\theta}^{n+1/3}_{N,\,j,k}. \qquad (17)$$

Sequentially applying the sweep method for *N-1* and *N-2* , we find $\tilde{\theta}^{n+1/3}_{N-1,\,j,k}$ and $\tilde{\theta}^{n+1/3}_{N-2,\,j,k}$:

$$\tilde{\theta}^{n+1/3}_{N-1,\,j,k} = \alpha_{N-1,\,j,k}\tilde{\theta}^{n+1/3}_{N,\,j,k} + \beta_{N-1,\,j,k}; \qquad (18)$$

$$\begin{aligned}\tilde{\theta}^{n+1/3}_{N-2,\,j,k} &= \alpha_{N-2,\,j,k}\tilde{\theta}^{n+1/3}_{N-1,\,j,k} + \beta_{N-2,\,j,k} = \\ &= \alpha_{N-2,\,j,k}\left(\alpha_{N-1,\,j,k}\tilde{\theta}^{n+1/3}_{N,\,j,k} + \beta_{N-1,\,j,k}\right) + \beta_{N-2,\,j,k} = \\ &= \alpha_{N-2,\,j,k}\alpha_{N-1,\,j,k}\tilde{\theta}^{n+1/3}_{N,\,j,k} + \alpha_{N-2,\,j,k}\beta_{N-1,\,j,k} + \beta_{N-2,\,j,k}.\end{aligned} \qquad (19)$$

Substituting $\tilde{\theta}^{n+1/3}_{N-1,\,j,k}$ and $\tilde{\theta}^{n+1/3}_{N-2,\,j,k}$ in (18), (19) instead of $\tilde{\theta}^{n+1/3}_{N-1,\,j,k}$ and $\tilde{\theta}^{n+1/3}_{N-2,\,j,k}$, in (17) we find $\tilde{\theta}^{n+1/3}_{N,\,j,k}$:

$$\alpha_{N-2,\,j,k}\alpha_{N-1,\,j,k}\mu\tilde{\theta}^{n+1/3}_{N,\,j,k} + \alpha_{N-2,\,j,k}\beta_{N-1,\,j,k}\mu + \beta_{N-2,\,j,k}\mu - 4\alpha_{N-1,\,j,k}\mu\tilde{\theta}^{n+1/3}_{N,\,j,k} -$$

$$-4\beta_{N-1,\,j,k}\mu + 3\mu\tilde{\theta}^{n+1/3}_{N,\,j,k} = 2\Delta x\, e_1\xi\theta_E - 2\Delta x\,\xi\tilde{\theta}^{n+1/3}_{N,\,j,k};$$

$$\tilde{\theta}^{n+1/3}_{N,\,j,k} = \frac{2\Delta x e_1\xi\theta_E - \left(\beta_{N-2,\,j,k} + \alpha_{N-2,\,j,k}\beta_{N-1,\,j,k} - 4\beta_{N-1,\,j,k}\right)\mu}{2\Delta x\xi + \left(\alpha_{N-2,\,j,k}\alpha_{N-1,\,j,k} - 4\alpha_{N-1,\,j,k} + 3\right)\mu}. \qquad (20)$$

In the reverse course of the sweep in successively decreasing order of index i , the values of concentrations $\tilde{\theta}^{n+1/3}_{N-1,\,j,k}$, $\tilde{\theta}^{n+1/3}_{N-2,\,j,k}$, ..., are determined $\tilde{\theta}^{n+1/3}_{0,\,j,k}$.

By applying the above procedures for sending Ox , *we* have the following:

$$\frac{\tilde{\theta}^{n+2/3}_{i,\,j,k} - \tilde{\theta}^{n+1/3}_{i,\,j,k}}{\Delta t/3} + \sigma_1\tilde{\theta}^{n+2/3}_{i,\,j,k} = \frac{\mu}{\Delta x^2}\left(\tilde{\theta}^{n+1/3}_{i+1,\,j,k} - 2\tilde{\theta}^{n+1/3}_{i,\,j,k} + \tilde{\theta}^{n+1/3}_{i-1,\,j,k}\right) +$$

$$+\frac{\mu}{\Delta y^2}\left(\tilde{\theta}^{n+2/3}_{i,\,j+1,k} - 2\tilde{\theta}^{n+2/3}_{i,\,j,k} + \tilde{\theta}^{n+2/3}_{i,\,j-1,k}\right) +$$

$$+\frac{1}{\Delta z^2}\left(\kappa_{k+0,5}\tilde{\theta}^{n+1/3}_{i,\,j,k+1} - \left(\kappa_{k-0,5} + \kappa_{k+0,5}\right)\tilde{\theta}^{n+1/3}_{i,\,j,k} + \kappa_{k-0,5}\tilde{\theta}^{n+1/3}_{i,\,j,k-1}\right) + \frac{1}{3}e_1\delta_{i,\,j,k}Q.$$

Opening the brackets and simplifying like terms, we finally get

$$\frac{\mu}{\Delta y^2}\tilde{\theta}^{n+2/3}_{i,\,j-1,k} - \left(\frac{3}{\Delta t} + \sigma_1 + \frac{2\mu}{\Delta y^2}\right)\tilde{\theta}^{n+2/3}_{i,\,j,k} + \frac{\mu}{\Delta y^2}\tilde{\theta}^{n+2/3}_{i,\,j+1,k} =$$

$$= -\left(\left(\frac{3}{\Delta t} - \frac{2\mu}{\Delta x^2} - \frac{\kappa_{k-0,5} + \kappa_{k+0,5}}{\Delta z^2}\right)\tilde{\theta}^{n+1/3}_{i,\,j,k} + \frac{\mu}{\Delta x^2}\tilde{\theta}^{n+1/3}_{i-1,\,j,k} + \frac{\mu}{\Delta x^2}\tilde{\theta}^{n+1/3}_{i+1,\,j,k} + \right.$$

$$\left. + \frac{\kappa_{k-0,5}}{\Delta z^2}\tilde{\theta}^{n+1/3}_{i,\,j,k-1} + \frac{\kappa_{k+0,5}}{\Delta z^2}\tilde{\theta}^{n+1/3}_{i,\,j,k+1} + \frac{1}{3}e_1\delta_{i,\,j,k}Q\right).$$

To simplify the above equation, we introduce the following notation:

$$\bar{a}_{i,\,j,k} = \frac{\mu}{\Delta y^2};\quad \bar{b}_{i,\,j,k} = \frac{3}{\Delta t} + \sigma_1 + \frac{2\mu}{\Delta y^2};\quad \bar{c}_{i,\,j,k} = \frac{\mu}{\Delta y^2};$$

$$\bar{d}_{i,\,j,k} = \left(\frac{3}{\Delta t} - \frac{2\mu}{\Delta x^2} - \frac{\kappa_{k-0,5} + \kappa_{k+0,5}}{\Delta z^2}\right)\tilde{\theta}^{n+1/3}_{i,\,j,k} + \frac{\mu}{\Delta x^2}\tilde{\theta}^{n+1/3}_{i-1,\,j,k} + \frac{\mu}{\Delta x^2}\tilde{\theta}^{n+1/3}_{i+1,\,j,k} +$$

$$+\frac{\kappa_{k-0,5}}{\Delta z^2}\tilde{\theta}^{n+1/3}_{i,\,j,k-1} + \frac{\kappa_{k+0,5}}{\Delta z^2}\tilde{\theta}^{n+1/3}_{i,\,j,k+1} + \frac{1}{3}e_1\delta_{i,\,j,k}Q.$$

As a result, we obtain a tridiagonal system of linear algebraic equations:

$$\bar{a}_{i,j,k}\tilde{\theta}^{n+2/3}_{i,j-1,k} - \bar{b}_{i,j,k}\tilde{\theta}^{n+2/3}_{i,j,k} + \bar{c}_{i,j,k}\tilde{\theta}^{n+2/3}_{i,j+1,k} = -\bar{d}_{i,j,k}. \quad (21)$$

Similarly, repeating the above procedure for the boundary condition (11), we obtain

$$-\mu\frac{-3\tilde{\theta}^{n+2/3}_{i,0,k} + 4\tilde{\theta}^{n+2/3}_{i,1,k} - \tilde{\theta}^{n+2/3}_{i,2,k}}{2\Delta y} = \xi e_1\theta_E - \xi\tilde{\theta}^{n+2/3}_{i,0,k};$$

$$3\mu\tilde{\theta}^{n+2/3}_{i,0,k} - 4\mu\tilde{\theta}^{n+2/3}_{i,1,k} + \mu\tilde{\theta}^{n+2/3}_{i,2,k} = 2\Delta y e_1\xi\theta_E - 2\Delta y\xi\tilde{\theta}^{n+2/3}_{i,0,k}; \quad (22)$$

From the resulting system of equations

$$\bar{a}_{i,1,k}\tilde{\theta}^{n+2/3}_{i,0,k} - \bar{b}_{i,1,k}\tilde{\theta}^{n+2/3}_{i,1,k} + \bar{c}_{i,1,k}\tilde{\theta}^{n+2/3}_{i,2,k} = -\bar{d}_{i,1,k}$$

we find $\tilde{\theta}^{n+2/3}_{i,2,k}$:

$$\tilde{\theta}^{n+2/3}_{i,2,k} = -\frac{\bar{a}_{i,1,k}}{\bar{c}_{i,1,k}}\tilde{\theta}^{n+2/3}_{i,0,k} + \frac{\bar{b}_{i,1,k}}{\bar{c}_{i,1,k}}\tilde{\theta}^{n+2/3}_{i,1,k} - \frac{\bar{d}_{i,1,k}}{\bar{c}_{i,1,k}}. \quad (23)$$

Substituting (23) instead $\tilde{\theta}^{n+2/3}_{i,2,k}$ of in (22), we find $\tilde{\theta}^{n+2/3}_{i,0,k}$:

$$\tilde{\theta}^{n+\frac{2}{3}}_{i,0,k} = \frac{4\mu\bar{c}_{i,1,k} - \bar{b}_{i,1,k}\mu}{3\mu\bar{c}_{i,1,k} - \bar{a}_{i,1,k}\mu + 2\Delta y\xi}\tilde{\theta}^{n+\frac{2}{3}}_{i,1,k} + \frac{\bar{d}_{i,1,k} + 2\Delta y\bar{c}_{i,1,k}e_1\xi\theta_E}{3\mu\bar{c}_{i,1,k} - \bar{a}_{i,1,k}\mu + 2\Delta y\xi}. \quad (24)$$

Using the above formulas (24), we find the values of the sweep coefficients $\bar{\alpha}_{i,0,k}$ and $\bar{\beta}_{i,0,k}$:

$$\bar{\alpha}_{i,0,k} = \frac{4\mu\bar{c}_{i,1,k} - \bar{b}_{i,1,k}\mu}{3\mu\bar{c}_{i,1,k} - \bar{a}_{i,1,k}\mu + 2\Delta y\xi};\quad \bar{\beta}_{i,0,k} = \frac{\bar{d}_{i,1,k} + 2\Delta y e_1\bar{c}_{i,1,k}\xi\theta_E}{3\mu\bar{c}_{i,1,k} - \bar{a}_{i,1,k}\mu + 2\Delta y\xi}. \quad (25)$$

In the boundary condition (11), using the approximation of the second order of accuracy, we obtain

$$\mu\frac{\tilde{\theta}^{n+2/3}_{i,M-2,k} - 4\tilde{\theta}^{n+2/3}_{i,M-1,k} + 3\tilde{\theta}^{n+2/3}_{i,M,k}}{2\Delta y} = \xi e_1\theta_E - \xi\tilde{\theta}^{n+2/3}_{i,M,k};$$

$$\mu\tilde{\theta}^{n+2/3}_{i,M-2,k} - 4\mu\tilde{\theta}^{n+2/3}_{i,M-1,k} + 3\mu\tilde{\theta}^{n+2/3}_{i,M,k} = 2\Delta y e_1\xi\theta_E - 2\Delta y\xi\tilde{\theta}^{n+2/3}_{i,M,k}. \quad (26)$$

Consistently applying the sweep method for *M-1* and *M-2* , we find $\tilde{\theta}^{n+2/3}_{i,M-1,k}$ and $\tilde{\theta}^{n+2/3}_{i,M-2,k}$:

$$\tilde{\theta}^{n+\frac{2}{3}}_{i,M-2,k} = \bar{\alpha}_{i,M-2,k}\tilde{\theta}^{n+\frac{2}{3}}_{i,M-1,k} + \bar{\beta}_{i,M-2,k} = \bar{\alpha}_{i,M-2,k}\left(\bar{\alpha}_{i,M-1,k}\tilde{\theta}^{n+2/3}_{i,M,k} + \bar{\beta}_{i,M-1,k}\right) +$$
$$+\bar{\beta}_{i,M-2,k} = \bar{\alpha}_{i,M-2,k}\bar{\alpha}_{i,M-1,k}\tilde{\theta}^{n+2/3}_{i,M,k} + \bar{\alpha}_{i,M-2,k}\bar{\beta}_{i,M-1,k} + \bar{\beta}_{i,M-2,k}. \quad (27)$$

Substituting $\tilde{\theta}^{n+2/3}_{i,M-1,k}$ and $\tilde{\theta}^{n+2/3}_{i,M-2,k}$ in (26) and (27), instead of $\tilde{\theta}^{n+2/3}_{i,M-1,k}$ and $\tilde{\theta}^{n+2/3}_{i,M-2,k}$ in (26) we find $\tilde{\theta}^{n+2/3}_{i,M,k}$:

$$\left(\bar{\alpha}_{i,M-2,k}\bar{\alpha}_{i,M-1,k}\mu - 4\bar{\alpha}_{i,M-1,k}\mu + 3\mu + 2\Delta y\xi\right)\tilde{\theta}^{n+2/3}_{i,M,k} =$$
$$= 2\Delta y e_1\xi\theta_E - \bar{\beta}_{i,M-2,k}\mu - \bar{\alpha}_{i,M-2,k}\bar{\beta}_{i,M-1,k}\mu + 4\bar{\beta}_{i,M-1,k}\mu;$$
$$\tilde{\theta}^{n+2/3}_{i,M,k} = \frac{2\Delta y e_1\xi\theta_E - \left(\bar{\beta}_{i,M-2,k} + \bar{\alpha}_{i,M-2,k}\bar{\beta}_{i,M-1,k} - 4\bar{\beta}_{i,M-1,k}\right)\mu}{2\Delta y\xi + \left(\bar{\alpha}_{i,M-2,k}\bar{\alpha}_{i,M-1,k} - 4\bar{\alpha}_{i,M-1,k} + 3\right)\mu}. \quad (28)$$

By reverse running, in the order of successively decreasing index j , the values of concentrations $\tilde{\theta}^{n+2/3}_{i,M-1,k}$, $\tilde{\theta}^{n+2/3}_{i,M-2,k}$,..., $\tilde{\theta}^{n+2/3}_{i,0,k}$ are in the following form:

$$\tilde{\theta}^{n+2/3}_{i,j,k} = \bar{\alpha}_{i,j,k}\tilde{\theta}^{n+2/3}_{i,j+1,k} + \bar{\beta}_{i,j,k};\qquad i = \overline{1,N-1},\quad j = \overline{M-1,0},\quad k = \overline{1,L-1}. \quad (29)$$

above procedures are applied for sending Oz and we have

$$\frac{\tilde{\theta}_{i,j,k}^{n+1}-\tilde{\theta}_{i,j,k}^{n+2/3}}{\Delta t/3}+\sigma_1\tilde{\theta}_{i,j,k}^{n+1}=\frac{\mu}{\Delta x^2}\left(\tilde{\theta}_{i+1,j,k}^{n+2/3}-2\tilde{\theta}_{i,j,k}^{n+2/3}+\tilde{\theta}_{i-1,j,k}^{n+2/3}\right)+$$

$$+\frac{\mu}{\Delta y^2}\left(\tilde{\theta}_{i,j+1,k}^{n+2/3}-2\tilde{\theta}_{i,j,k}^{n+2/3}+\tilde{\theta}_{i,j-1,k}^{n+2/3}\right)+$$

$$+\frac{1}{\Delta z^2}\left(\kappa_{k+0,5}\tilde{\theta}_{i,j,k+1}^{n+1}-\left(\kappa_{k-0,5}+\kappa_{k+0,5}\right)\tilde{\theta}_{i,j,k}^{n+1}+\kappa_{k-0,5}\tilde{\theta}_{i,j,k-1}^{n+1}\right)+\frac{1}{3}e_1\delta_{i,j,k}Q.$$

Opening the brackets and simplifying like terms, we finally get

$$\frac{\kappa_{k-0,5}}{\Delta z^2}\tilde{\theta}_{i,j,k-1}^{n+1}-\left(\frac{3}{\Delta t}+\sigma_1+\frac{\kappa_{k-0,5}+\kappa_{k+0,5}}{\Delta z^2}\right)\tilde{\theta}_{i,j,k}^{n+1}+\frac{\kappa_{k+0,5}}{\Delta z^2}\tilde{\theta}_{i,j,k+1}^{n+1}=$$

$$=-\left(\left(\frac{3}{\Delta t}-\frac{2\mu}{\Delta x^2}-\frac{2\mu}{\Delta y^2}\right)\tilde{\theta}_{i,j,k}^{n+2/3}+\frac{\mu}{\Delta x^2}\tilde{\theta}_{i-1,j,k}^{n+2/3}+\frac{\mu}{\Delta x^2}\tilde{\theta}_{i+1,j,k}^{n+2/3}+\right.$$

$$\left.+\frac{\mu}{\Delta y^2}\tilde{\theta}_{i,j-1,k}^{n+2/3}+\frac{\mu}{\Delta y^2}\tilde{\theta}_{i,j+1,k}^{n+2/3}+\frac{1}{3}e_1\delta_{i,j,k}Q\right).$$

To simplify the equations obtained, we introduce the notation:

$$\bar{\bar{a}}_{i,j,k}=\frac{\kappa_{k-0,5}}{\Delta z^2};\qquad \bar{\bar{b}}_{i,j,k}=\frac{3}{\Delta t}+\sigma_1+\frac{\kappa_{k-0,5}+\kappa_{k+0,5}}{\Delta z^2};\quad \bar{\bar{c}}_{i,j,k}=\frac{\kappa_{k+0,5}}{\Delta z^2};$$

$$\bar{\bar{d}}_{i,j,k}=\left(\frac{3}{\Delta t}-\frac{2\mu}{\Delta x^2}-\frac{2\mu}{\Delta y^2}\right)\tilde{\theta}_{i,j,k}^{n+2/3}+\frac{\mu}{\Delta x^2}\tilde{\theta}_{i-1,j,k}^{n+2/3}+\frac{\mu}{\Delta x^2}\tilde{\theta}_{i+1,j,k}^{n+2/3}+$$

$$+\frac{\mu}{\Delta y^2}\tilde{\theta}_{i,j-1,k}^{n+2/3}+\frac{\mu}{\Delta y^2}\tilde{\theta}_{i,j+1,k}^{n+2/3}+\frac{1}{3}e_1\delta_{i,j,k}Q^{n+1},$$

and we obtain a tridiagonal system of linear algebraic equations:

$$\bar{\bar{a}}_{i,j,k}\tilde{\theta}_{i,j,k-1}^{n+1}-\bar{\bar{b}}_{i,j,k}\tilde{\theta}_{i,j,k}^{n+1}+\bar{\bar{c}}_{i,j,k}\tilde{\theta}_{i,j,k+1}^{n+1}=-\bar{\bar{d}}_{i,j,k}. \tag{30}$$

For the boundary condition (12) at $z = 0$ we obtain

$$-\kappa_1\frac{-3\tilde{\theta}_{i,j,0}^{n+1}+4\tilde{\theta}_{i,j,1}^{n+1}-\tilde{\theta}_{i,j,2}^{n+1}}{2\Delta z}=\beta\tilde{\theta}_{i,j,0}^{n+1}-fe_1;$$

$$3\kappa_1\tilde{\theta}_{i,j,0}^{n+1}-4\kappa_1\tilde{\theta}_{i,j,1}^{n+1}+\kappa_1\tilde{\theta}_{i,j,2}^{n+1}=2\Delta z\beta\tilde{\theta}_{i,j,0}^{n+1}-2e_1\Delta zf. \tag{31}$$

From the resulting system of equations

$$\bar{\bar{a}}_{i,j,1}\tilde{\theta}_{i,j,0}^{n+1}-\bar{\bar{b}}_{i,j,1}\tilde{\theta}_{i,j,1}^{n+1}+\bar{\bar{c}}_{i,j,1}\tilde{\theta}_{i,j,2}^{n+1}=-\bar{\bar{d}}_{i,j,1},$$

we find $\tilde{\theta}_{i,j,2}^{n+1}$:

$$\tilde{\theta}_{i,j,2}^{n+1}=-\frac{\bar{\bar{a}}_{i,j,1}}{\bar{\bar{c}}_{i,j,1}}\tilde{\theta}_{i,j,0}^{n+1}+\frac{\bar{\bar{b}}_{i,j,1}}{\bar{\bar{c}}_{i,j,1}}\tilde{\theta}_{i,j,1}^{n+1}-\frac{\bar{\bar{d}}_{i,j,1}}{\bar{\bar{c}}_{i,j,1}}. \tag{32}$$

Substituting (32) instead $\tilde{\theta}_{i,j,2}^{n+1}$ of in (31), we find $\tilde{\theta}_{i,j,0}^{n+1}$:

$$3\kappa_1\tilde{\theta}_{i,j,0}^{n+1}-4\kappa_1\tilde{\theta}_{i,j,1}^{n+1}-\frac{\bar{\bar{a}}_{i,j,1}}{\bar{\bar{c}}_{i,j,1}}\kappa_1\tilde{\theta}_{i,j,0}^{n+1}+\frac{\bar{\bar{b}}_{i,j,1}}{\bar{\bar{c}}_{i,j,1}}\kappa_1\tilde{\theta}_{i,j,1}^{n+1}-\frac{\bar{\bar{d}}_{i,j,1}}{\bar{\bar{c}}_{i,j,1}}\kappa_1=$$

$$=2\Delta z\beta\tilde{\theta}_{i,j,0}^{n+1}-2e_1\Delta zf;$$

$$\tilde{\theta}_{i,j,0}^{n+1}=\frac{4\kappa_1\bar{\bar{c}}_{i,j,1}-\bar{\bar{b}}_{i,j,1}\kappa_1}{3\kappa_1\bar{\bar{c}}_{i,j,1}-\bar{\bar{a}}_{i,j,1}\kappa_1-2\Delta z\beta}\tilde{\theta}_{i,j,1}^{n+1}+\frac{\bar{\bar{d}}_{i,j,1}\kappa_1+2e_1\Delta z\bar{\bar{c}}_{i,j,1}f}{3\kappa_1\bar{\bar{c}}_{i,j,1}-\bar{\bar{a}}_{i,j,1}\kappa_1-2\Delta z\beta}. \tag{33}$$

Using the above formulas (33), we find the values of the sweep coefficients $\bar{\bar{\alpha}}_{i,j,0}$ and $\bar{\bar{\beta}}_{i,j,0}$:

$$\bar{\bar{\alpha}}_{i,j,0} = \frac{4\kappa_1 \bar{\bar{c}}_{i,j,1} - \bar{\bar{b}}_{i,j,1}\kappa_1}{3\kappa_1 \bar{\bar{c}}_{i,j,1} - \bar{\bar{a}}_{i,j,1}\kappa_1 - 2\Delta z \beta}; \qquad \bar{\bar{\beta}}_{i,j,0} = \frac{\bar{\bar{d}}_{i,j,1}\kappa_1 + 2e_1 \Delta z \bar{\bar{c}}_{i,j,1} f}{3\kappa_1 \bar{\bar{c}}_{i,j,1} - \bar{\bar{a}}_{i,j,1}\kappa_1 - 2\Delta z \beta}. \tag{34}$$

In the boundary condition (12) for $z = H_z$, using the approximation of the second order of accuracy, we obtain the following:

$$\kappa_L \frac{\tilde{\theta}^{n+1}_{i,j,L-2} - 4\tilde{\theta}^{n+1}_{i,j,L-1} + 3\tilde{\theta}^{n+1}_{i,j,L}}{2\Delta z} = \xi e_1 \theta_E - \xi \tilde{\theta}^{n+1}_{i,j,L};$$

$$\kappa_L \tilde{\theta}^{n+1}_{i,j,L-2} - 4\kappa_L \tilde{\theta}^{n+1}_{i,j,L-1} + 3\kappa_L \tilde{\theta}^{n+1}_{i,j,L} = 2\Delta z e_1 \xi \theta_E - 2\Delta z \xi \tilde{\theta}^{n+1}_{i,j,L}. \tag{35}$$

Sequentially applying the sweep method for *L-1* and *L-2*, we find $\tilde{\theta}^{n+1}_{i,j,L-1}$ and $\tilde{\theta}^{n+1}_{i,j,L-2}$:

$$\tilde{\theta}^{n+1}_{i,j,L-1} = \bar{\bar{\alpha}}_{i,j,L-1}\tilde{\theta}^{n+1}_{i,j,L} + \bar{\bar{\beta}}_{i,j,L-1}; \tag{36}$$

$$\tilde{\theta}^{n+1}_{i,j,L-2} = \bar{\bar{\alpha}}_{i,j,L-2}\tilde{\theta}^{n+1}_{i,j,L-1} + \bar{\bar{\beta}}_{i,j,L-2} = \bar{\bar{\alpha}}_{i,j,L-2}\left(\bar{\bar{\alpha}}_{i,j,L-1}\tilde{\theta}^{n+1}_{i,j,L} + \bar{\bar{\beta}}_{i,j,L-1}\right) + \bar{\bar{\beta}}_{i,j,L-2} = \bar{\bar{\alpha}}_{i,j,L-2}\bar{\bar{\alpha}}_{i,j,L-1}\tilde{\theta}^{n+1}_{i,j,L} + \bar{\bar{\alpha}}_{i,j,L-2}\bar{\bar{\beta}}_{i,j,L-1} + \bar{\bar{\beta}}_{i,j,L-2}. \tag{37}$$

Substituting $\tilde{\theta}^{n+1}_{i,j,L-1}$ and $\tilde{\theta}^{n+1}_{i,j,L-2}$ in (36) and (37), instead of $\tilde{\theta}^{n+1}_{i,j,L-1}$ and $\tilde{\theta}^{n+1}_{i,j,L-2}$ in (35) we find $\tilde{\theta}^{n+1}_{i,j,L}$:

$$\tilde{\theta}^{n+1}_{i,j,L} = \frac{2\Delta z e_1 \xi \theta_E - \left(\bar{\bar{\beta}}_{i,j,L-2} + \bar{\bar{\alpha}}_{i,j,L-2}\bar{\bar{\beta}}_{i,j,L-1} - 4\bar{\bar{\beta}}_{i,j,L-1}\right)\kappa_L}{2\Delta z \xi + \left(\bar{\bar{\alpha}}_{i,j,L-2}\bar{\bar{\alpha}}_{i,j,L-1} - 4\bar{\bar{\alpha}}_{i,j,L-1} + 3\right)\kappa_L}. \tag{38}$$

By reverse running in successively decreasing order of the index k, the values of the concentrations of harmful substances $\tilde{\theta}^{n+1}_{i,j,L-1}$, $\tilde{\theta}^{n+1}_{i,j,L-2}$,..., $\tilde{\theta}^{n+1}_{i,j,0}$ are in the following form:

$$\tilde{\theta}^{n+1}_{i,j,k} = \bar{\bar{\alpha}}_{i,j,k}\tilde{\theta}^{n+1}_{i,j,k+1} + \bar{\bar{\beta}}_{i,j,k}; \quad i = \overline{1, N-1}, \quad j = \overline{1, M-1}, \quad k = \overline{L-1, 0}. \tag{39}$$

Thus, in this section, an efficient numerical algorithm is developed for solving the problem (1)-(6) with the second order of accuracy in space variables. By software implementation of the developed model and algorithm, it is possible to carry out SE on a computer to study and predict the ecological state of the industrial regions under consideration [34].

4. About programming complexes

The software tool presented in this paragraph, developed in the Embarcadero environment Rad Studio version 10 in C# programming language, consists of: main control unit; block for entering input data; computational unit for numerical integration and conducting SE; a block for forming a database based on the results of numerical calculations; block for interpreting the results of numerical calculations in the form of graphic, animation and tabular objects (Fig. 1-4).

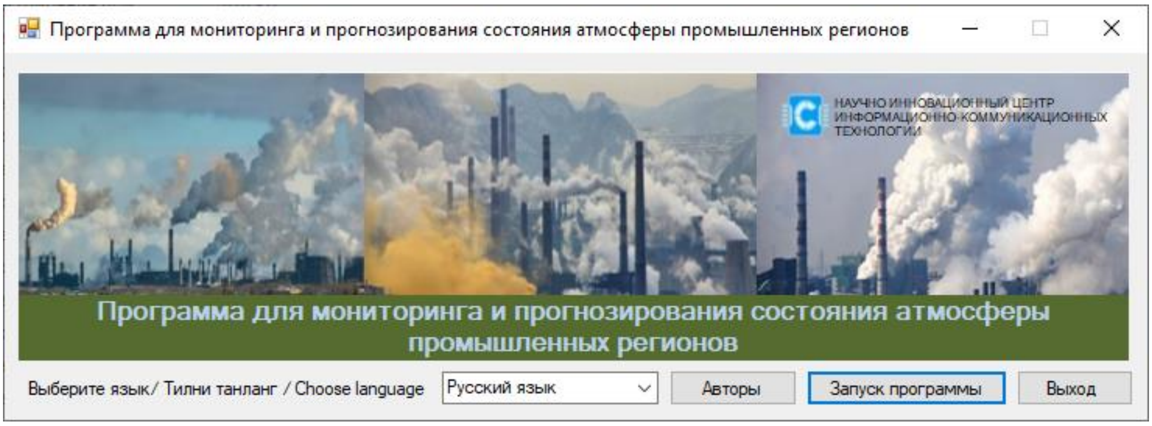


Fig. 1. Main program launch window

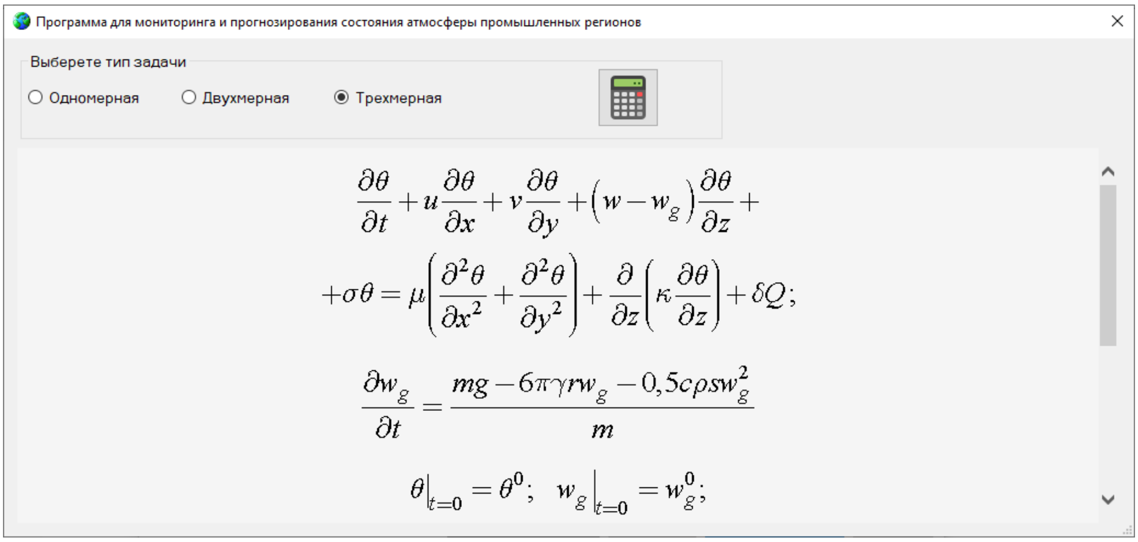


Fig. 2. Interface for selecting the language locale, model and, accordingly, the algorithm of the problem being solved

As part of a software and instrumental complex, this program allows you to monitor the dynamic distribution of harmful emissions in the atmosphere on a computer monitor depending on weather and climate data, terrain orography, characteristics of the underlying surface of the earth and other disturbances affecting the process as a whole [35].

Functional properties of the program:

- creation of an information model for calculating the concentration of harmful substances;
- calculation of the concentration of harmful substances in the atmosphere of the region, depending on the power of the located sources and their number, weather and climatic conditions and other additional factors;
- monitoring and forecasting the state of the air basin of the region under consideration and an animated presentation of the results of numerical calculations performed on a computer;
- activating the interface mode in order to maximize the visualization of human-machine interaction.

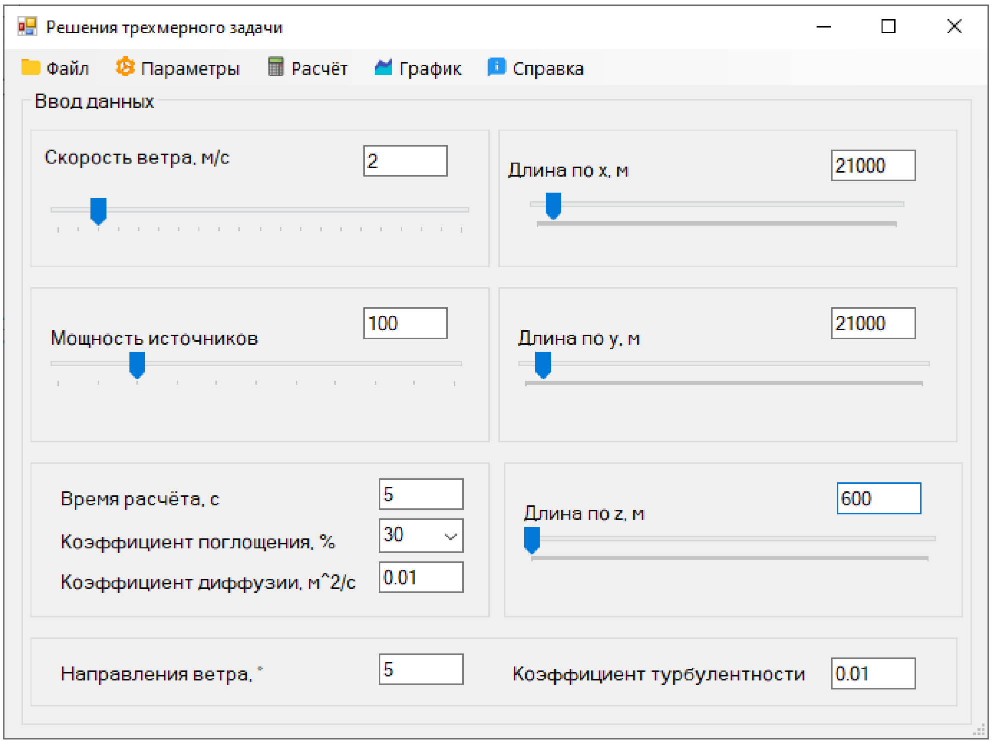
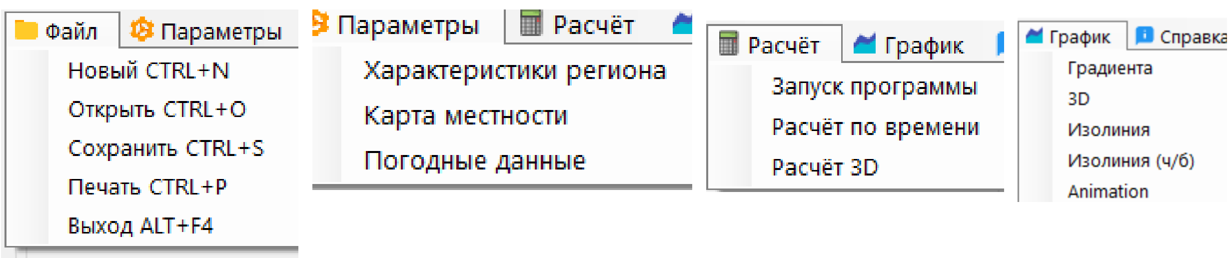


Fig. 3. Interface for entering the parameters of the problem to be solved

The window in fig. 3 contains such parameters as wind speed, source power, calculation time, absorption coefficient of harmful substances in the atmosphere, diffusion coefficient, turbulence coefficient and the size of the area under consideration.



a) *b*) *c*) *d*)

FIg. 4. Elements of the main menu of the program

The main menu of the program (Fig. 4) allows the user to perform the following actions:

In the "File" menu (Fig. 4 *a*), the user has the opportunity to create a new CE; save the created experiment configuration and settings; open a previously saved configuration and experiment parameters; printing of results and graphic objects; and the end of the program.

The "Parameters" menu (Fig. 4 *b*) provides the ability to set the parameters and conditions of the problem being solved: characteristics of the region (size, relief), geographical map of the region (map details), as well as meteorological data for a given area.

Using the "Calculation" menu (Fig. 4 *c*), the user can start the calculation procedure in accordance with the specified settings.

The "Graph" menu (Fig. 4 *d*) is designed to select the preferred form of visualization of the calculation results, including in the form of a gradient, 3D diagram (Fig. 5), color or black and white isolines (Fig. 6), as well as an animated image .

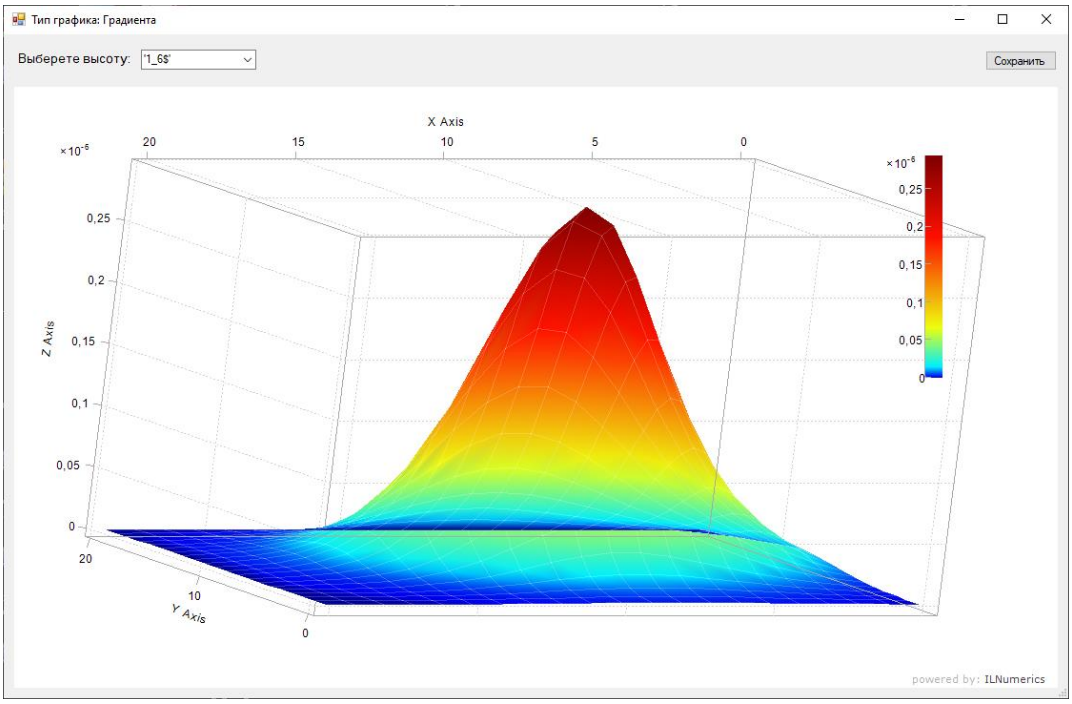


Fig. 5. An example of visualization of calculation results in the form of a color gradient of a three-dimensional shape

As shown in fig. 5, any available layer can be selected for visualization according to the height of the problem solution area. In addition, when choosing a real geographical map of the region, the visualized layers with the distribution of the concentration of harmful substances can be superimposed on the original map - the substrate in the form of a translucent layer, or can be exported for use in third-party applications.

The practical application of the developed software tool in solving the problems of monitoring and predicting the ecological state of the atmosphere makes it possible to automate calculations to determine the concentration of pollution in the boundary layer of the atmosphere, taking into account real meteorological parameters and characteristics of the terrain, to visualize the scale and geography of the distribution of harmful emissions in the environment for a visual assessment of possible negative effects on the ecological state of adjacent territories, taking into account the maximum permissible sanitary standards, as well as to support decision-making on environmental protection measures.

To calibrate the models developed and implemented in this program, a series of SEs was carried out on test data on the problem of predicting the spread of harmful substances in the atmosphere, taking into account the particle settling rate (Fig. 6).

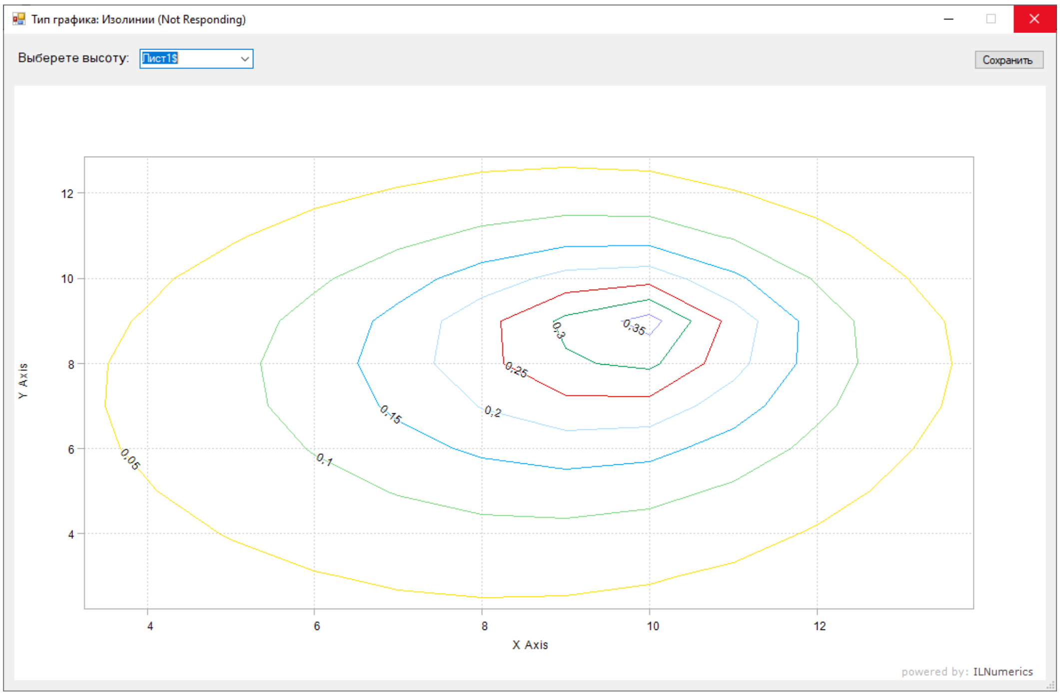


Fig. 6. Distribution of the concentration of aerosol particles with a northeast wind (225°), wind speed $u = 3$ **m/s and absorption coefficient** $\sigma = 30\%$

The SE were also carried out by setting various parameters of the process of transfer and diffusion of harmful substances, which are given in (Fig. 7-10).

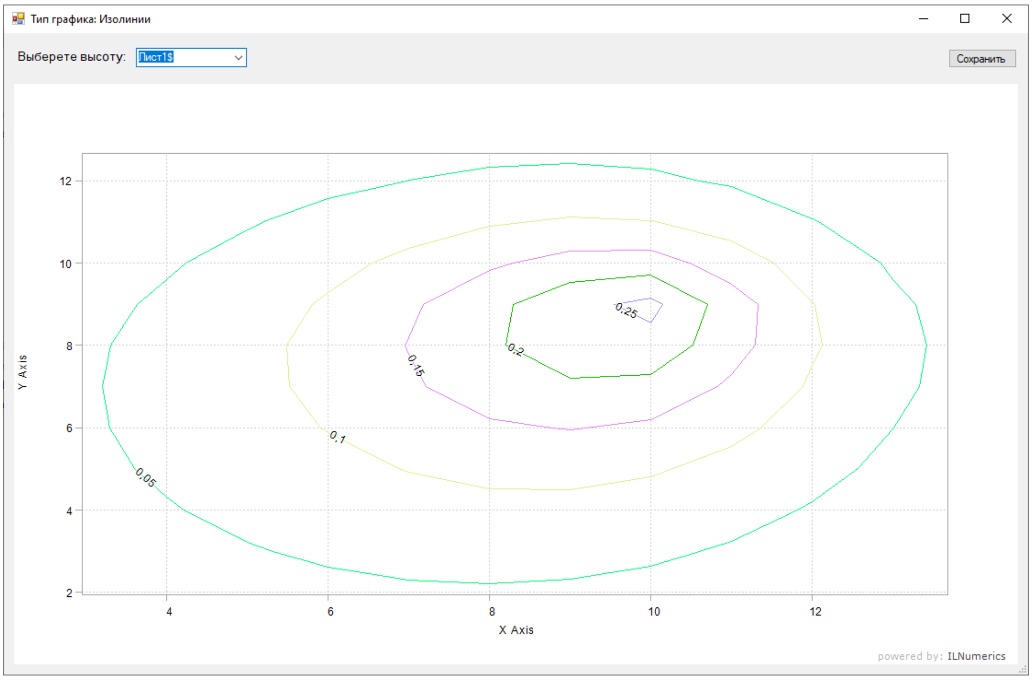


Fig. 7. Distribution of the concentration of aerosol particles with a northeast wind (225°), wind speed $u = 4$ **m/s and absorption coefficient** $\sigma = 30\%$

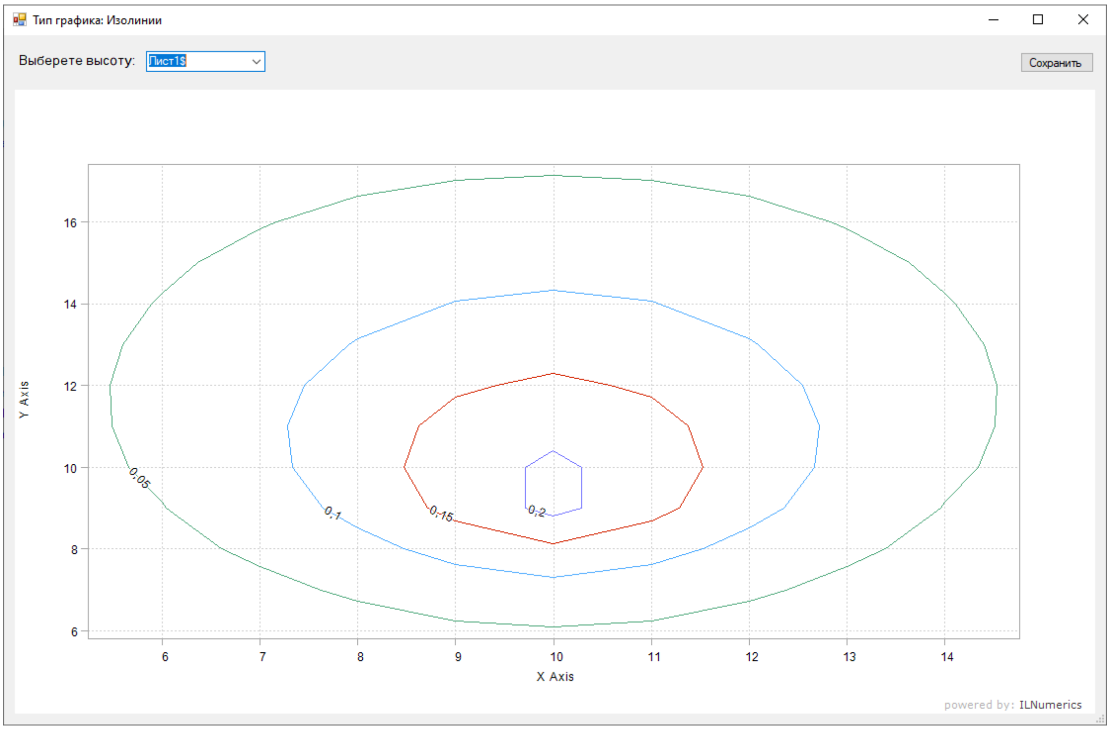


Fig. 8. Distribution of the concentration of aerosol particles at the south wind (0°), wind speed $u = 5$ m/s and absorption coefficient $\sigma = 30\%$

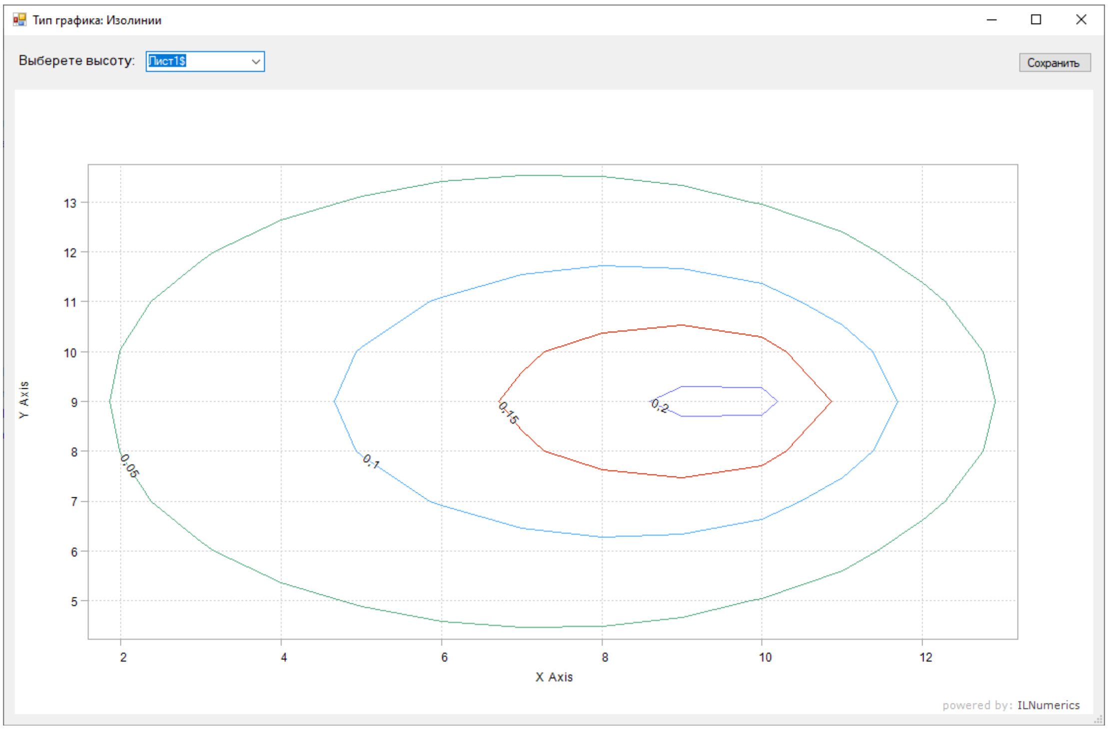


Fig. 9. Distribution of the concentration of aerosol particles with east wind (180°), wind speed $u = 5$ m/s and absorption coefficient $\sigma = 30\%$

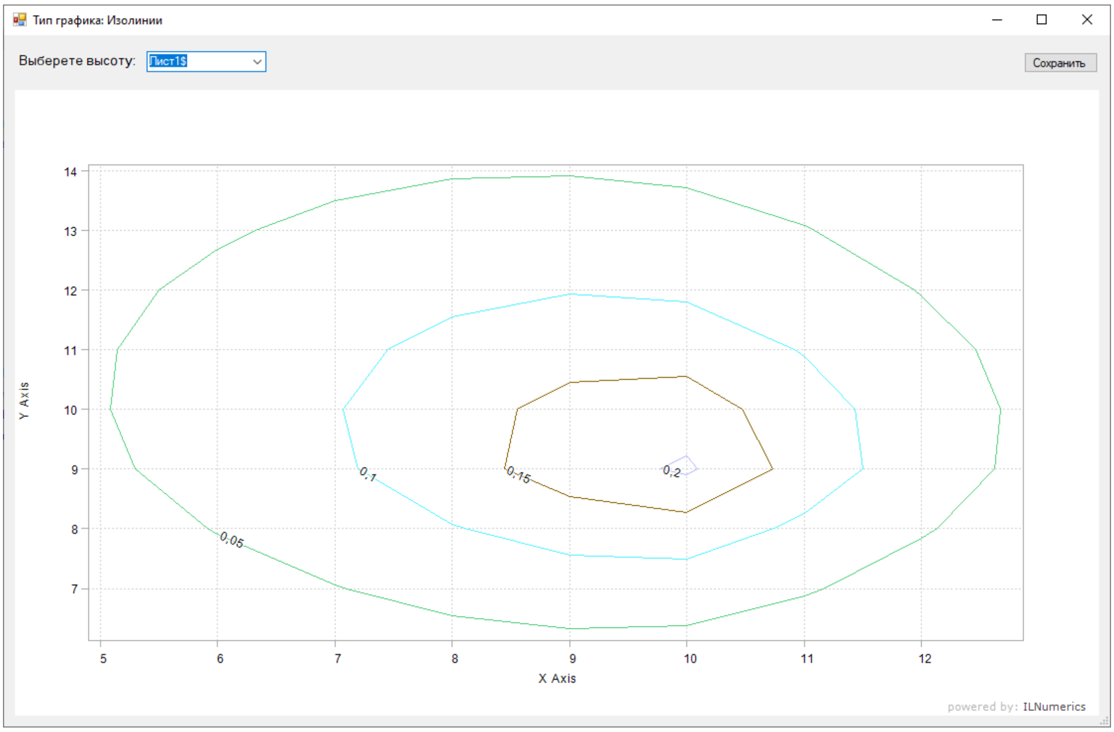


Fig. 10. The concentration of aerosol particles under the prevailing east wind with a speed of $u = 4$ m/s and an absorption coefficient $\sigma = 40\%$

Numerical experiments were carried out for various values of the turbulence coefficient, ground roughness, horizontal and vertical wind speeds, etc. [36].

The transfer was carried out uniformly at all levels, depending on the direction of the wind speed. From the theory of the boundary layer, it is known that with unstable stratification, the values $\kappa(z)$ grow to a level of 200–400 m and rapidly fall with height, tending to zero at the upper boundary of the boundary layer (the height of the boundary layer with unstable stratification reaches 1000–1600 m), and with stable stratification $\kappa(z)$ grow insignificantly in the surface layer and fall with height (the height of the boundary layer is 400–600 m) [36].

According to the numerical calculations carried out on a computer, with an increase in the horizontal component of the wind speed, aerosol particles emitted from industrial sources are transported in the direction of the wind. The area of distribution of pollutants in the surface layer of the atmosphere expands with an increase in the speed of the air mass of the atmosphere (Fig. 6, Fig. 8). This is especially observed at H = 200- 300 м. [36, 21].

Another parameter that significantly affects the process under study is the absorption coefficient (Fig. 9-10). The results of computational experiments show that 10-18% of aerosol particles are absorbed by the atmosphere. The increase in the absorption of harmful substances depends on the humid state of the air mass, and the change in the coefficient itself directly depends on the temperature and humidity of the atmosphere [37].

Special attention was paid to a parameter that also significantly affects the change in the concentration of harmful substances in the atmosphere and on the earth's surface - the rate of deposition of harmful particles, which depends both on the vertical component of the wind speed and on the physical and mechanical properties of the particles [34].

References:

1. Smirnov, N.N., Nikitin, V.F., Legros, J.C., & Shevtsova, V.M. (2002). Motion and Sedimentation of Particles in Turbulent Atmospheric Flows above Sources of Heating. *Aerosol Science & Technology*, vol. 36, Issue 2, pp. 101-122.
2. Saxby, J., Beckett, F., Cashman, K., Rust, A., & Tennant, E. (2018). The impact of particle shape

on fall velocity: Implications for volcanic ash dispersion modelling. *Journal of Volcanology and Geothermal Research*, 2018, vol. 362, pp. 32-48.

3. Naslund, E., & Thaning, L. (1991). On the Settling Velocity in a Nonstationary Atmosphere. *Aerosol Science and Technology*, vol. 14, Issue 2, pp. 247-256.
4. Iversen, T., & Nordeng, T.E. (2001). A numerical model suitable for the simulation of a broad class of circulation systems on the atmospheric mesoscale. *Norwegian Institute for Air Research Techn. Rep*, № 2, pp. 38-51.
5. Holden, H., Hvistendahl, K., & Lie, K. (2000). Operator splitting methods for degenerate convection-diffusion equations II: numerical examples with emphasis on reservoir simulation and sedimentation. *Computational Geosciences*, vol. 4, № 4, pp. 287-322.
6. Geiser, J., & Kravvaritis, Ch. (n.d.). A Domain Decomposition method based on iterative Operator Splitting method. Applied Numerical Mathematics, 209, vol. 59, pp. 608-623.
7. Havasi, A., Bartholy, J., & Farago, I. (2001). Splitting method and its application in air pollution modeling. *Idojaras*, vol. 105, № 1, pp. 39-58.
8. Kurbonov, N. (2018). *Numerical algorithm for solving the problem of gas filtration in porous media by the method of coordinate splitting.* Actual problems of mathematical modeling, algorithmization and programming: Proceedings of the Republican Scientific and Practical Conference. September 17-18, 2018, (pp.133-136). Tashkent.
9. Vallero, D. (2014). *Fundamentals of Air Pollution*, 5th ed, Academic Press, 996 p.
10. Burnett R., et.al. (2018). Global estimates of mortality associated with longterm exposure to outdoor fine particulate matter. *Proceedings of the National Academy of Sciences*, vol. 115, № 38, pp. 9592-9597.
11. Moore, M. (2014). *China's 'airpocalypse' kills 350,000 to 500,000 each year. The Telegraph*. Retrieved from https://bit.ly/2svFHCw
12. Zubkova, A.D. (2018). *Assessment of the impact of pollutant emissions from stationary and diffuse sources on abiotic and biotic components of urban ecosystems:* Cand. cand. chem. Sciences. (p.146). Kazan.
13. Gurova, O.S. (2013). Basic principles of classification of sources of air pollution in urban areas of the Southern Federal District. *Naukovedenie*, No. 5, pp. 1-8. https://naukovedenie.ru/PDF/11trgsu513.pdf
14. Bauer, S., Tsigaridis, K., & Miller, R. (2016). Significant atmospheric aerosol pollution caused by world food cultivation. *Geophysical Research Letters*, vol. 43, Issue 10, pp. 5394-5400, DOI: 10.1002/2016GL068354.
15. (2016). *Outdoor Air Pollution*. International Agency for Research on Cancer, Lion: IARC, 2016, (IARC Monographs. Vol. 109), 448 p.
16. (2013). *Analytical information on the state of the natural environment in Uzbekistan for 2009*. Ecological Movement of Uzbekistan, Retrieved from https://bit.ly/38IkeXL
17. (2011). *List of maximum permissible concentrations (MPC) of pollutants in the atmospheric air of populated areas on the territory of the Republic of Uzbekistan*. Hygienic standards: SanPiN RUz N 0293-11, Instead of SanPiN RUz N 0179-04; approved May 16, 2011, (p.33). Tashkent. (Sanitary rules and norms, hygienic standards of the Republic of Uzbekistan).
18. Sharipov, D., Muradov, F., & Akhmedov, D. (2019). Numerical Modeling Method for Short-Term Air Quality Forecast in Industrial Regions. *Applied Mathematics E-Notes*, № 19, pp. 575-584.
19. Ravshanov, N., Muradov, F., & Akhmedov, D. (2018). Mathematical software to study the harmful substances diffusion in the atmosphere. *Ponte*, vol. 74, № 8/1, pp. 171-179, DOI: 10.21506/j.ponte.2018.8.13
20. Ravshanov, N., Muradov, F., & Akhmedov, D. (2020). Operator splitting method for numerical solving the atmospheric pollutant dispersion problem. *Journal of Physics: Conference Series.* London, Vol. 1441. 012164.
21. Muradov, F.A., Rakhimov, A.Z., Nuriev, Kh.U., & Turdibekov, B.B. (2019). Mathematical model and computational experiment for predicting the ecological state of the atmospheric boundary layer. *Teoriya i praktika sovremennoy nauki*, No. 2 (44), pp. 264-270.
22. Muradov, F.A., & Ravshanov, Z.N. (2019). *Modeling the process of transfer and diffusion of fine aerosols in the atmosphere, taking into account the particle settling rate.* Innovative ideas, developments and modern problems of their application in production, as well as in education: Proceedings of the International Scientific and Practical Conference. Part 2. April 15, 2019, (pp.132-134). Andijan: ASU.
23. Muradov, F.A. (2019). *Numerical modeling of the spread of harmful substances taking into account the rate of deposition of particles in the atmosphere.* Modern problems and their solutions of information and communication technologies and telecommunications: Proceedings of the Republican Scientific and Technical Conference. Part 3. May 30-31, 2019, (pp. 118-121). Ferghana.
24. Muradov, F.A. (2019). *Numerical modeling of the spread of toxic substances taking into*

account the rate of deposition of particles into the surface layer of the atmosphere. Current state and prospects for the use of information technologies in management: Proceedings of the Republican Scientific and Technical Conference. September 5, 2019, (pp.172-178). Samarkand.

25. Ravshanov, N., & Muradov, F.A. (2017). Model and numerical algorithm for the process of transfer and diffusion of active fine particles in the atmosphere. *Information Technologies of Modeling and Control*, Voronezh: Scientific book, No. 2 (104), pp. 132-142.
26. Muradov, F., & Akhmedov, D. (1915). *Numerical Modeling of Atmospheric Pollutants Dispersion Taking Into Account Particles Settling Velocity*. IEEE International Conference on Information Science and Communications Technologies (ICISCT), 4-6 Nov. 2019, Tashkent, 2019, pp. 1-5, Retrieved from https://doi.org/10.1109/ICISCT47635.2019.9011915
27. Ravshanov, N., & Sharipov, D.K. (2011). Model and numerical algorithm for predicting the process of distribution of harmful substances in the atmosphere. *Uzb. journal "Problems of Informatics and Energy*", Tashkent, No. 4, pp. 18-25.
28. Ravshanov, N., Serikbaev, B., & Serikbaeva, E. (2010). Methodology for protecting ecosystems from pollution sources. *Stiinta Agricola, Chisinau*, No. 1, pp. 68-73.
29. Ravshanov, N., Sharipov, D.K., & Toshtemirova, N. (2015). Computer modeling of the propagation of aerosol emissions in the atmosphere. *Problems of Computational and Applied Mathematics*, Tashkent, No. 1, pp. 16-27, http://goo.gl/75uzYF
30. Ravshanov, N., Muradov, F., & Akhmedov, D.D. (2019). *Mathematical and software support for forecasting the ecological state of the atmosphere of industrial regions*. Informatics: problems, methodology, technologies: Proceedings of the XIX International Scientific and Methodological Conference. February 14-15, 2019, (pp.148-153). Voronezh: Voronezh State University.
31. Muradov, F.A., & Narzikulov, Z.Kh. (2018). *Mathematical and software for forecasting and monitoring the spread of toxic substances in the atmosphere.* Actual issues of production of import-substitute products based on the use of local raw materials in the regions of the Fergana Valley: Proceedings of the International Conference, (pp.168-171). Namangan.
32. Ravshanov, N., Muradov, F., & Akhmedov, D. (2018). *Air pollution modeling in spherical coordinates.* Aktual`nye problemy matematicheskogo modelirovanija, algoritmizacii i programmirovanija: Materialy Respublikanskoj nauchno-prakticheskoj konferencii. 17-18 sentjabrja 2018, (pp.210-216). Tashkent: Fan va ta#lim poligraf.
33. Ravshanov, N., Muradov, F., & Akhmedov, D. (2018). *Numerical modeling of the process of transfer and diffusion of harmful substances in the atmosphere in a spherical coordinate system.* Problems of optimization of complex systems: Proceedings of the XIV International Asian School-Seminar. July 20-31, 2018, In 2 hours - Almaty, 2018, Part 2, pp. 142-151.
34. Sharipov, D.K., Muradov, F., & Ravshanov, Z.N. (2017). Mathematical model and computational experiment for monitoring and forecasting the ecological state of the atmospheric boundary layer. *Problems of Computational and Applied Mathematics*, Tashkent, No. 6 (12), pp. 15-28.
35. Muradov, F.A., & Mirbabaev, Sh.R. (2020). *Creating a software tool for predicting and monitoring the spread of harmful substances in the atmosphere.* Innovative ideas, developments into practice: problems and solutions: Proceedings of the International scientific-practical online conference. May 27-28, 2020, (pp.65-68). Andijan.
36. Ravshanov, N., Karshiev, D.A., & Yuldashev, B.E. (2018). Modeling the process of transfer and diffusion of fine particles in the atmosphere, taking into account soil erosion. *International Journal of Humanities and Natural Sciences*, T. 4, pp. 140-152.
37. Muradov, F.A., & Ravshanov, Z.N. (2018). *Mathematical model and computational experiment for forecasting the ecological state of the atmosphere in regions*. Informatics: problems, methodology, technologies: Proceedings of the 18th International Conference. February 8-9, 2018, (pp.191-195). Voronezh, 2018, V. 5.