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NATURAL SCIENTIFIC ASPECTS OF GRAPHIC SOLUTION OF PHYSICAL AND MATHEMATICAL PROBLEMS ON THE EDUCATIONAL TOPIC «MECHANICAL OSCILLATIONS» WHEN PREPARING GRADUATES FOR THE USE (ON THE BASIS OF «GROWTH POINTS» ASKINO SECONDARY SCHOOL №1 REPUBLIC OF BASHKORTOSTAN)

Abstract: *The article considers a graphical method for solving physical and mathematical problems for mechanical vibrations in the subject "Physics" in a secondary school. A combined approach is given for constructing graphs of the dependence of the main physical quantities of a harmonic oscillation on time in one plane (mathematical Cartesian coordinate system). The mechanism for constructing graphs of displacement, speed, acceleration, restoring force, kinetic and potential energy of mechanical vibrations using computer technology is shown. As an example, a physical problem for a harmonic oscillation has been solved, taking into account mathematical methods for plotting function graphs. The proposed method is of great theoretical and practical importance for subject teachers, teachers in the preparation of graduates of general education schools for the USE in physics, mathematics, computer science and ICT. All experiments, calculations and plotting were carried out on modern computer equipment equipped with the "Growth Points" line.*

Key words: *training, education, physics, mathematics, computer science, USE, graph, function, dependence, oscillations, waves, mechanical, harmonic, displacement, speed, acceleration, force, energy, kinetic, potential, period, phase, amplitude, material, point, stiffness, oscillatory circuit, middle school.*

Language: *English*

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Introduction

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At the present stage of education and upbringing in a rural general education school, the system of lesson models is a certain technology for building and designing the educational process as a whole. At the same time, regardless of the type of educational institution, the profile of education, the level of teaching, each lesson model in any subject has been and remains specific to this day [1].

However, the boundaries of its applicability are determined by the experience and wisdom of the teacher, the conditions of a given school, a separate class, the content of the examination papers of the Unified State Examination (USE).

The high-quality preparation of a graduate for the Unified State Examination in subjects of the natural science block largely depends on the correct methodology for building the educational process when using new educational programs within the framework of the Federal State Educational Standard (FSES), as well as the use of specific methodological tools for organizing educational activities.

For example, according to [2], when developing the content of Control and Measuring Materials (CMM), only some elements of the content of one of the most important sections of the general course of physics in the school *"Mechanical vibrations"*, *"Waves"*, *"Sound"*.

Although, the author of [3], while analyzing the tasks of the official demonstration versions of the Unified State Examination in physics of past years, offers a list of topics that cause the greatest difficulty for graduates in these sections. These are tasks №№ 3, 4, 5, 18.

We have found that, in general, the tasks of the demonstration versions of the USE in physics are limited only to considering the dependence of the displacement on time $x = x(t)$, and in real tasks of the USE, there are quite often graphs to identify the dependence of the speed on time $\upsilon = \upsilon(t)$, acceleration versus time $a = a(t)$, force versus time $F = F(t)$, kinetic energy versus time $W_{\text{kin}} = W_{\text{kin}}(t)$, and potential energy versus time $W_{\text{pot}} = W_{\text{pot}}(t)$.

2. The purpose of the study. In this regard, this article proposes methodological recommendations and practical methods for solving physical problems for numerical calculations and plotting the dependences of the parameters of mechanical oscillation in a projection connection.

All necessary measurements, physical experiments, mathematical calculations, computer experiments were carried out on the basis of the Digital and Humanitarian Profile *"Growth Point"* Askino Secondary School №1 Republic of Bashkortostan [4].

3. Background of the issue. It is important for a modern teacher of mathematics and physics to know the background of the origin of the doctrine of oscillations, which will allow students to convey more accurate historical information and facts from the life of famous scientists, which is rarely found in the author's educational textbooks on these subjects.

The well-known scientists G. Galileo and H. Huygens began to study mechanical vibrations for the first time in the middle of the 17th century [5].

Galileo established the independence of the period from the amplitude (isochronisms) of small fluctuations $T = T(A)$, observing the swinging of the chandelier in the cathedral and measuring the time of the pulse on his hand, fixing all the data.

Huygens invented the first clock with a pendulum back in 1657, and during the publication of the monograph *"Pendulum Clock"* (1673), known to our time, he investigated a number of problems associated with the movement of the pendulum, in particular, he found the center of swing of a physical pendulum.

It was the studies of pendulum oscillations, undertaken by the Italian scientist G. Galileo, and then by the Dutch scientist H. Huygens, that played an important role in the emergence of classical mechanics.

The study of electromagnetic oscillations at the end of the 19th century by the English physicist W. Thomson was of great importance for understanding oscillatory processes in modern physics [6]. A lot of information and results about oscillations are also contained in the works of the English physicist J. Rayleigh.

The invention of radio by A.S. Popov (1985) was one of the main doctrines of Russian physics about oscillations and the most important technical application of electromagnetic oscillations in modern life [7].

4. Methodical methods of presenting the theory of the question. In addition to historical facts, a special place is occupied by the theoretical preparation of graduates for the exam in physics. Without knowing the theory of the oscillation process, it is impossible to correctly solve the proposed exam problems.

As our observations and work experience show, the examinees are simply confused in elementary terms and concepts. When preparing graduates for the USE, a teacher of physics needs to explain that, by definition [8], oscillations are processes that repeat exactly or approximately at regular intervals.

Depending on the physical nature, mechanical, electromagnetic and other types of oscillations are distinguished.

Many students do not immediately understand the nature of oscillations. In this regard, it is important for a teacher of mathematics and physics to focus on the motivational stage of the lesson, giving real life examples. For example, high-rise buildings, clock pendulums oscillate high-voltage wires oscillate under the action of the wind, a car on springs, and many other similar cases.

On the other hand, we can say that sound is the same fluctuations in the density and pressure of air, there are also radio waves, a beating pulse, or an earthquake like ground vibrations and other examples from life.

When studying individual topics on fluctuations from year to year, students should understand that despite the different nature of fluctuations, the same patterns are found in them, they are described by the same mathematical equations [9], they are studied by common methods, the development and application of which constitute the task of the theory of oscillations.

It is important for a modern teacher to single out harmonic oscillations from all the various types of oscillations, since they play the main and essential role in solving most USE tasks in both mathematics and physics, where the oscillating value changes over time according to the sine law or cosine.

Being USE experts in checking tasks of increased complexity of past years, we noticed that teachers of physics and mathematics need to learn certain algorithms for solving problems of increased complexity, only after that they teach their wards to correctly solve problems for fluctuations and graphs of functions.

Therefore, we offer our own, implemented in training and tested for many years, combined method of graphical representation, solving the most typical problems for mechanical vibrations, having previously formulated a physical and mathematical problem with the introduction of basic concepts.

5. Practical methods of presenting the theory of the issue. Let mechanical vibrations mean repeated deviations of a material point (body) with mass *m* from the equilibrium position under the action of forces of various natures [10]. Then the main sign by which it is possible to distinguish oscillatory motion from other types of motion will be its periodicity.

From this it follows that the period of oscillation *T* will be such a period of time during which the body makes one complete oscillation. In turn, the number of oscillations per unit time will be the frequency of oscillations $\nu = 1 / T$.

As a rule, free mechanical vibrations are inherent in a system that is brought out of its original state of equilibrium by means of some kind of influence, and then left to itself. Often this action is a force F proportional to the displacement modulus $|x|$ and directed towards the equilibrium position $F = k \cdot |x|$, where $k = const$.

Such a force is called a restoring force [11], under the influence of which the body performs free harmonic oscillations along the abscissa axis.

Combining the analyzed works [10] and [11], we can say that the displacement of the body point coordinate at any time t is described by laws of the form: $x = A \cdot sin(\omega \cdot t + \varphi) = A \cdot cos(\omega \odot t + \varphi_0)$, where $A = x_{max}$ is the largest deviation (amplitude) of a point (body) from the equilibrium position (meter), $\omega = 2 \cdot \pi \odot \nu$ is the cyclic oscillation frequency (Hz), ν is the natural frequency of oscillations (Hz), $\varphi = \varphi_0 + \pi / 2$ is the initial phase of oscillations at time $t=0$.

To identify the dependence of the speed υ and acceleration a on time t, it is necessary to refer to the course of algebra and the beginning of analysis. According to [12], the instantaneous speed is the derivative of the coordinate with respect to time, i.e. $\upsilon_x(t) = x'(t) = - A \cdot \omega \cdot sin(\omega \odot t + \varphi) = \upsilon_{max} \cdot sin(\omega \odot t + \varphi) = \upsilon_{max} \cdot cos(\omega \cdot t + \varphi_0 + \pi / 2)$.

In turn, acceleration is the derivative of the speed with respect to time, or the second derivative of the coordinate with respect to time [13], i.e. $a_x(t) = x''(t) = - A \odot \omega^2 \cdot cos(\omega \cdot t + \varphi_0) = a_{max} \cdot cos(\omega \cdot t + \varphi) = \upsilon_{max} \cdot cos(\omega \cdot t + \varphi_0)$.

For further plotting of dependency graphs, the teacher needs to bring to the attention of graduates that the following expressions of the form $x_{max} = A$, $\upsilon_{max} = - A \cdot \omega$, $a_{max} = - A \cdot \omega^2$ work in expressions for displacement, speed and acceleration.

It is important that each student learns to *"read"* the proposed function graphs. To do this, the teacher, when preparing a graduate for the exam, needs to show that on the combined graphs (Figure 1) of the dependencies $x(t)$, $\upsilon(t)$, $a(t)$, the speed is ahead of the phase shift fluctuations by an angle $(\pi / 2)$, which also means a shift by a quarter of the period T.

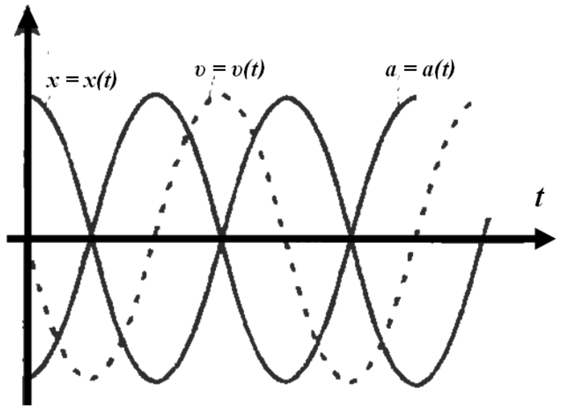

Figure 1 – Graphics of displacement, speed and acceleration versus time

It should also be noted that acceleration fluctuations occur in antiphase with fluctuations in point displacements from the equilibrium position.

On the other hand, based on Newton's II law, it follows that the restoring force $F = m \cdot a = - m \cdot \omega^2 \cdot A \cdot cos(\omega \cdot t + \varphi_0) = - m \cdot \omega^2 \cdot x = - k \cdot x$, where $k = m \cdot \omega^2$.

To build graphs of the dependence of oscillations of kinetic and potential energy, it is necessary to take into account that the period of harmonic oscillations of a material point (body) is an expression of the form: $T = 2\pi\sqrt{m/k}$.

Given this expression, the kinetic energy of an oscillating material point (body) as a result of transformations is equal to the expression of the following form: $W_{kin} = (m \cdot \upsilon^2) / 2 = [m \cdot \omega^2 \cdot A^2 \cdot cos^2(\omega \cdot t + \varphi_0)] / 2$.

In turn, the potential energy of an oscillating material point (body), displaced from the equilibrium position by x, is determined by an expression of the form: $W_{pot} = (k \cdot x^2) / 2 = [m \cdot \omega^2 \cdot A^2 \cdot cos^2(\omega \cdot t + \varphi_0)] / 2$.

However, from [13], using the reduction formula $cos^2\alpha = [1 + cos(2 \cdot \alpha)] / 2$, the potential energy can be rewritten in a different, modified form: $W_{pot} = [k \cdot A^2 \cdot cos^2(\omega \cdot t + \varphi_0)] / 2 = k \cdot A^2 \cdot [1 + cos2(\omega \cdot t + \varphi_0)] / 4$.

Therefore, we believe that only after all the transformations carried out, it is possible to analyze the graphs of the constructed dependencies.

Figure 2 shows that the potential energy, like the displacement, changes with time strictly according to the harmonic law. Here it is important to convey to students that the frequency is twice the frequency of the displacement oscillations. Moreover, fluctuations in kinetic energy occur in antiphase with fluctuations in potential energy.

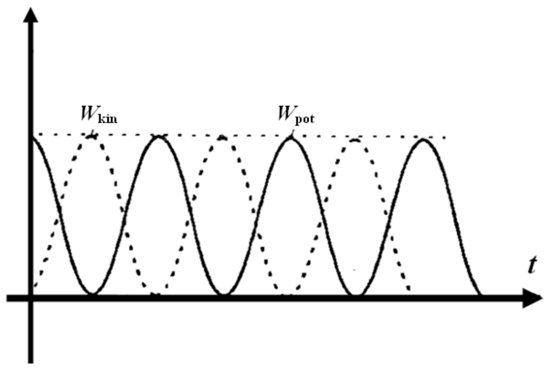

Figure 2 – Graphics of kinetic and potential energy versus time

Then the total energy W of a material point (body) oscillating without damping will be constant and equal to the sum of the maximum values of the kinetic and potential energies, i.e. we get an expression of the following form: $W = W_{kin} + W_{pot} = (m\upsilon^2_{max})/2 = (kx^2_{max})/2$.

Usually, in real USE tasks, in contrast to the tasks of the demonstration version, a mathematical pendulum [14] is considered as a material point located at the end of an inextensible weightless thread of length l. Therefore, the teacher needs to show the graduates that at deflection angles within $5–8^o$, the pendulum oscillations are considered harmonic with a period equal to $T = 2\pi\sqrt{l/g}$, where g is the free fall acceleration.

Sometimes examinees are faced with problems on a spring pendulum [15], which is considered as a physical body with a mass m, attached to a spring with a stiffness k.

Therefore, here, when preparing students for the exam, the teacher must explain that, within certain limits, according to Hooke's law, the restoring force of the spring elasticity caused by deformation is directly proportional to the magnitude of the deformation. This means that the spring pendulum, as well as the mathematical pendulum, performs harmonic oscillations, the oscillation period of which is determined by the law $T = 2\pi\sqrt{m/k}$.

Summing up the current result of the above, we believe that this method proposes to teach a graduate of a general education school, in preparation for the USE, the correct graphical representation of changes in such physical quantities, for example, as displacement, speed, acceleration, force (directed to the equilibrium position), kinetic energy, potential energy, total energy of an oscillating material point. As a result, the examinee will not have problems in making the correct solution to the proposed problem.

For example, in task №17 of the demonstration version of the USE-2023 in physics [2], it is proposed to establish a correspondence between the graphs and physical quantities, the dependences of which on time these graphs can represent (images with graphs are given in the archive with the USE tasks).

Meanwhile, this task is good grounds for studying electromagnetic oscillations, solving problems on this topic, graphical representation of changes in physical quantities that directly characterize the electromagnetic oscillations themselves, for example, task №21 of the demonstration version of the USE-2023 of the same source [2].

We believe that any ability of a graduate of a secondary school to solve the task of the USE makes his knowledge effective and practically applicable in any non-standard situation, both at school and in exams and subsequent studies at a university.

Therefore, the combined method presented by us for solving problems and graphical representation of changes in mathematical and physical quantities during mechanical vibrations has its own properties and advantages.

(1) Firstly, the proposed method is designed to solve any problem of a certain level of complexity from a given type of tasks of the same type (mass character) in mathematics, physics, computer science and ICT.

(2) Secondly, the method is a strictly prescribed and defined sequence of actions and steps to be performed (determinism), i.e. solution algorithm (in physics), transformation mechanism (in mathematics), program code in a certain programming language (in computer science and ICT).

(3) Finally, thirdly, the solution of any problem from this class of problems provides the examinee with an accurate, reliable result (effectiveness).

6. Computer experiment on plotting function graphs in solving physical and mathematical problems. However, in collections of problems in physics by quite well-known authors, such as G.N. Stepanova [16] and A.P. Rymkevich [17], used in our general educational organization, in the section *"Mechanical vibrations and waves"*, there is still no task for the combined graphical representation of speed, acceleration, restoring force and other physical quantities characterizing an oscillating material point (body).

Therefore, the student, under the guidance of his teacher, has to *"finish"* the condition of the problem, and, accordingly, bring it to the final decision on all points of the above algorithm.

In this regard, we present the correct version of the practical solution and graphic design of one of the problems according to the above physical and mathematical methodology.

Consider problem №497 of the source [16] on the topic *"Mechanical vibrations and waves"*. According to the condition, a weight of *500 g* is suspended on a spring with a stiffness of *40 N/m*. It is necessary to plot the oscillations of this load if the amplitude is *1 cm*.

Having written all the given parameters through the accepted physical notation with the conversion of units of measurement to the *SI* system, we get that $k = 40$ N/m, $m = 500$ g $= 0{,}5$ kg, $x_{max} = A = 1$ cm $= 0{,}01$ m. According to the described theory, it is necessary to plot the dependences of displacement $x = x(t)$, speed $\upsilon = \upsilon(t)$, acceleration $a = a(t)$, restoring force $F = F(t)$, kinetic $W_{kin} = W_{kin}(t)$ and potential $W_{pot} = W_{pot}(t)$ of energy versus time t.

Before starting specific numerical calculations, let the time $t = 0$. Then the displacement $x = x_{max} = A = 0{,}01$ m. We obtain the dependence of the following form $x = x_{max} \cdot cos(\omega \cdot t)$ at $\varphi_0 = 0$.

Let us calculate the value of the cyclic oscillation frequency ω through the expression $k = m \cdot \omega^2$, that is, $\omega = \sqrt{k/m} = \sqrt{40/0{,}5} = \sqrt{80}$ Hz.

Only now it is possible to write down the expression for the displacement dependence $x = x(t)$ in a specific form of the form $x(t) = 0{,}01 \cdot cos(\sqrt{80} \cdot t) \approx 0{,}01 \cdot cos(8{,}944 \cdot t)$.

Let us find the dependence of the speed on time $\upsilon = \upsilon(t) = x'(t)$ as the first derivative of the displacement, we obtain $\upsilon(t) = -0{,}01 \cdot \sqrt{80} \cdot sin(\sqrt{80} \cdot t) \approx -0{,}089 \cdot sin(8{,}944 \cdot t)$.

Similarly, we write the expression for acceleration versus time $a = a(t) = \upsilon'(t) = x''(t)$ as the first derivative of the speed or, according to [13], the second derivative of the displacement. We get $a(t) = -0{,}01 \cdot 80 \cdot cos(\sqrt{80} \cdot t) \approx -0.8 \cdot cos(8{,}944 \cdot t)$.

According to Newton's II law [18], we obtain the time dependence of the force $F = F(t) = m \cdot a = 0{,}5 \cdot [-0{,}8 \cdot cos(\sqrt{80} \cdot t)] \approx -0{,}4 \cdot cos(8{,}944 \cdot t)$.

It remains to calculate the numerical values of the kinetic and potential energies. With correct calculations, they should be equal, i.e. $W_{kin} = (m \cdot \upsilon^2) / 2 = [40 \cdot (0{,}01)^2] / 2 = 0{,}002\ J$, in turn, $W_{pot} = (k \cdot x^2) / 2 = [0{,}5 \cdot (0{,}01)^2] / 2 = 0{,}002\ J$.

To build graphs of all dependencies, it is necessary to calculate the oscillation period T equal to

$$T = 2\pi\sqrt{m/k} = 2 \cdot 3{,}14 \cdot \sqrt{0{,}5/40} \approx 0{,}702\ s.$$

Thus, it can be seen that the period of oscillations of all physical quantities x, υ, a, F is the same and equal to $T \approx 0{,}702\ s$, and the period of change of the kinetic and potential energies is two times less than $T = T/2 = 0{,}351\ s$.

This means that the kinetic and potential energies periodically change from zero to the same maximum value with a frequency twice the frequency of the displacement oscillations. It is important not to forget and to remind students that the vibrations of both energies occur in antiphase to each other.

In turn, the total energy of the oscillating body remains constant and is equal to the sum of the maximum values of the kinetic and potential energies.

Figures 3-8 show graphs of all dependencies in one plane, built in the *Microsoft Office Excel* software application, which is studied in detail in the course of computer science and ICT [19, 20] of any secondary educational organization.

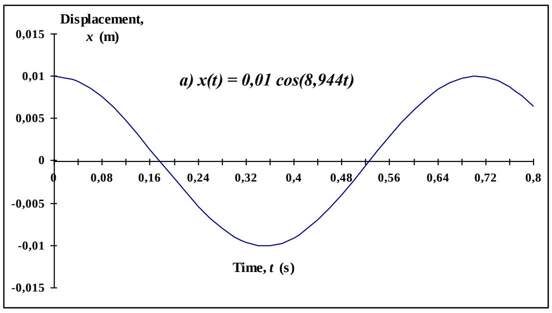

Figure 3 – Graph of displacement versus time

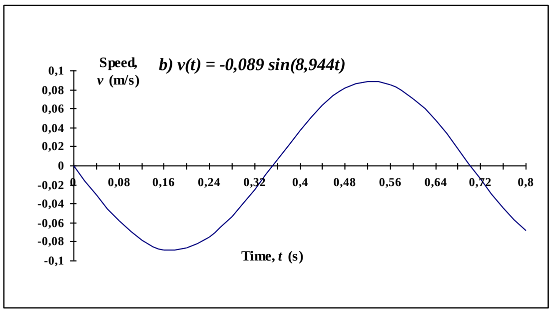

Figure 4 – Graph of speed versus time

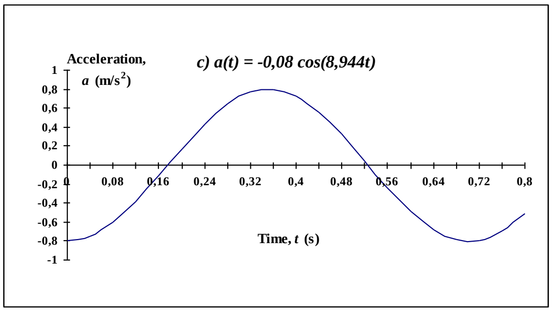

Figure 5 – Graph of acceleration versus time

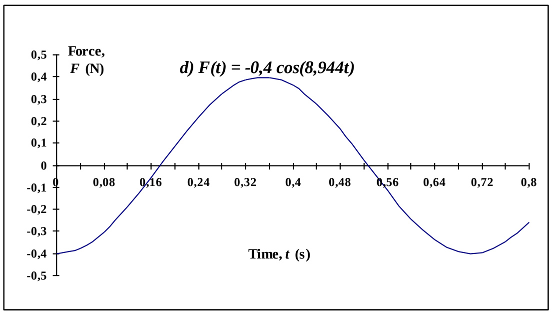

Figure 6 – Graph of force versus time

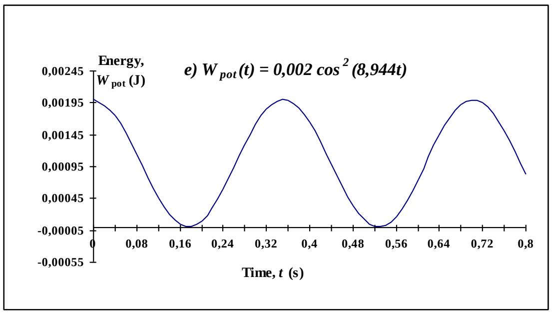

Figure 7 – Graph of potential energy versus time

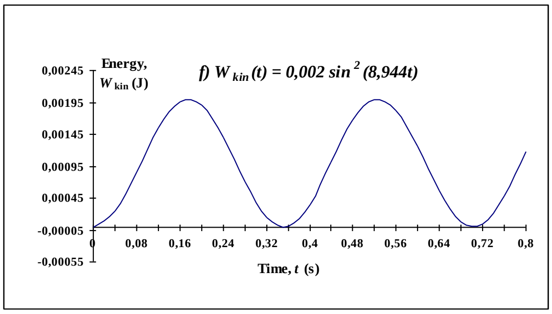

Figure 8 – Graph of kinetic energy versus time

As can be seen from the six dependencies obtained, the graphs of changes in physical quantities over time are in many cases convenient for describing and understanding any processes in more depth, for example, when studying processes occurring in an ideal oscillatory circuit, alternating electric current and other situations.

Therefore, a clear separation of the nature of the phenomena under consideration, their uniform description gives a huge advantage in the formation of the necessary skills for a modern graduate in preparation for the USE in physics, mathematics, computer science and ICT.

When selecting the content of lessons on the educational topic *"Mechanical vibrations"*, it is important for a modern teacher to equally use both theoretical and experimental methods for studying each phenomenon, to show their interconnection, interdependence and complementarity.

In conclusion, summarizing all of the above, we believe that it is not so much the study of ideal oscillatory systems given in school textbooks of physics, mathematics and problem books for them that plays a fundamentally important role, but a deep study of real systems in problems of increased complexity already on a real exam.

Ultimately, only with the help of the laws of modern science (theoretical and experimental), the whole world around us is known.

Our research and experiments on educational topics in the field of natural sciences are ongoing, new results and scientific articles are expected.

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