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FORMULATION OF A THREE-DIMENSIONAL (3D) BOUNDARY PROBLEM OF AN UNSTEADY WAVE PROCESS GENERATED IN THE SCHEMATIZED WATER RESERVOIR AND ANALYTICAL SOLUTIONS USING THE FINITE INTEGRAL TRANSFORM TECHNIQUE

***Abstract**: The work deals with a case, when there occurs the inflow (or outflow) of landslide masses, mud flow or water stream from all sides of the water reservoir at the definite velocity in the finite time interval, downstream (on the edges) and on the near shore slopes.*

***Key words**: Water reservoir, Dam, Mud flow, Landslide, Wave, Hydraulic structure.*

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Introduction

In order to provide a reliable operation of the hydraulic structure located in the water reservoir some engineering prediction calculations related to the impact of landslide masses or mud flows on the hydraulic structure have to be made. This process implies determination of not only dynamic effect directly on the structure, but also taking into account of those hydrodynamic forces action, which are originated in the form of waves and hydrodynamic pressures resulting from landslide inflow to the reservoir and its propagation.

The problem is formulated based on the small amplitude wave theory assuming that the reservoir water is ideal, incompressible and its movement is potential one. In this case the problem statement lies in the solution of Laplace equation $\Delta\varphi = 0$, in case of corresponding initial and boundary conditions, where $\varphi(x, y, z, t)$ is a velocity potential and $\Delta = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$ is a laplacian operator.

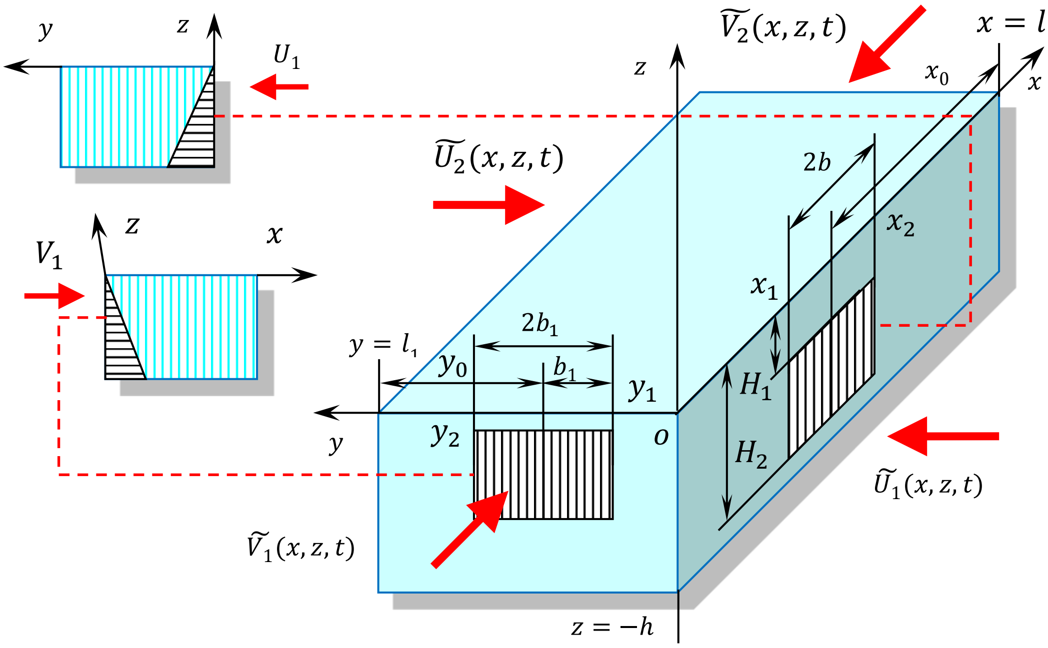

Fig. 1. General computational model of three-dimensional (3D) unsteady wave process generated resulting from landslide inflow to the water reservoir

Based on schematization, the water reservoir (or its section) is represented in the form of right-angle parallelogram with sizes: l – length, l_1 – width and h – depth, as is seen at the Figure 1.

A general case is considered, when takes place inflow (or outflow) of landslide masses, mud flow or water stream from all sides of a reservoir at a definite velocity and in a finite time interval. These velocities downstream are $\widetilde{V}_1(z,y,t)$ and$\widetilde{V}_2(z,y,t)$, while at the near shore slopes - $\widetilde{U}_1(z,x,t)$ and $\widetilde{U}_2(z,x,t)$. So, a formulated problem has the following shape:

Equation of continuity

$$\frac{\partial^2\varphi}{\partial x^2}+\frac{\partial^2\varphi}{\partial y^2}+\frac{\partial^2\varphi}{\partial z^2}=0 \qquad (1)$$

Boundary condition at the bottom

$$\left.\frac{\partial\varphi}{\partial z}\right|_{z=-h}=0 \qquad (2)$$

Boundary condition at the water reservoir boards

$$\left.\frac{\partial\varphi}{\partial x}\right|_{x=0}=\widetilde{V}_1(z,y,t), \qquad -\left.\frac{\partial\varphi}{\partial x}\right|_{x=l}=\widetilde{V}_2(z,y,t) \qquad (3)$$

Boundary condition at the water reservoir sides

$$\left.\frac{\partial\varphi}{\partial y}\right|_{y=0}=\widetilde{U}_1(z,x,t), \qquad -\left.\frac{\partial\varphi}{\partial y}\right|_{y=l_1}=\widetilde{U}_2(z,x,t) \qquad (4)$$

Free-surface condition

$$\left.\frac{1}{g}\frac{\partial^2\varphi}{\partial t^2}\right|_{z=0}+\left.\frac{\partial\varphi}{\partial z}\right|_{z=0}=0 \qquad (5)$$

Amplitude

$$\eta=-\left.\frac{1}{g}\frac{\partial\varphi}{\partial t}\right|_{z=0} \qquad (6)$$

It is meant that the mentioned velocities are originated within a certain time period. For instance, velocity $\widetilde{V}_1$ is determined within a time interval$[S_1,S_2]$:

$$\widetilde{V}_1(z,y,t)=\begin{cases}0 & ,t<S_1\\ V_1(z,y,t) & ,t\in[S_1,S_2]\\ 0 & ,t>S_2\end{cases} \qquad (7)$$

The analytical solutions of the three-dimensional boundary problem of the above-mentioned unsteady hydro dynamical (wave) motion in a general way was obtained by Professor T. Gvelesiani using the integral transform technique, so we no more touch upon a question of its solving.

Inflow of landslide/mud flow mainly takes place from one side only, that is why we discuss a special case of application of the mentioned formula, when the landslide mass inflow to the reservoir occurs at one section ($x_1 \le x \le x_2$) of the near shore slope ($y = 0$) only. Respectively, the landslide width along a side is $2b$. At that, $x_1 = l - x_0 - b$ and $x_2 = l - x_0 + b$, where x_0 is a distance from landslide section midpoint to the dam cross-section $x = l$. Inflow layer thickness (depth) h_0 varies within a following range $-H_1 < h_0 < -H_2$, where H_1 and H_2 are the coordinates of upper and lower limits of the inflow bed.

Landslide mass inflow velocity may be represented as follows:

$$\widetilde{U}_1(z,x,t)=U_1F_1(z)F_2(x)f_1(t) \qquad (8)$$

Let us consider a time interval $0 \le t \le t_0$, i.e. a case, when according to the above-mentioned notation, the values of S_1 and S_2 equal $S_1 = 0$ and

$S_2 = t_0$, respectively. Let us assume in this particular case that the functions entering the expression (8) can be approximated in such a way:

$$F_1(z) = \begin{cases} 0 & , -H_1 < z < 0 \\ 1 & , -H_2 \le z \le -H_1 \\ 0 & , -h \le z \le -H_2 \end{cases} \quad (9)$$

$$F_2(x) = \begin{cases} 0 & , x < x_1 \\ 1 & , x_1 \le x \le x_2 \\ 0 & , x > x_2 \end{cases} \quad (10)$$

$$f_1(t) = \begin{cases} 0 & , \mathrm{t} < 0 \\ 1 & , 0 \le t \le t_0 \\ 0 & , \mathrm{t} > \mathrm{t}_0 \end{cases} \quad (11)$$

Resulting from solution of the mentioned particular boundary problem the velocity potential is expressed as

$$\varphi(x,y,z,t) = \frac{1}{4}\varphi_0^0 + \frac{1}{2}\sum_{m=1}^{\infty}\varphi_0^m cosa_m y + \frac{1}{2}\sum_{n=1}^{\infty}\varphi_n^0 cosa_n x + \sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\varphi_n^m cosa_m y \cdot cosa_n x \quad (12)$$

Where

$$\varphi_n^m = \psi_n^m\left([S_n^m(z)\cdot f_1(t) - B_n^m(z)\cdot i_n^m(t)]\cdot J_n^m - \frac{G_n^m f_1(t)}{a_n^m}\right) \quad (13)$$

$$\varphi_0^0 = \psi_0^0\big([z\cdot f_1(t) - g\cdot i_0^0(t)]J_0^0 - G_0^0\cdot f_1(t)\big)$$

$$\psi_n^m = -\frac{4U_1}{ll_1\cdot a_n}(sina_n x_2 - sina_n x_1) \quad (14)$$

$$\psi_0^0 = -\frac{4U_1}{ll_1}(x_2 - x_1)$$

$$i_n^m(t) = \frac{2}{\gamma_n^m}sin\gamma_n^m\frac{t_0}{2}\cdot sin\gamma_n^m\left(t - \frac{t_0}{2}\right) \quad (15)$$

$$i_0^0(t) = t\cdot t_0 - \frac{t_0^2}{2}$$

$$J_n^m = -\frac{1}{a_n^m}[sha_n^m(h - H_1) - sha_n^m(h - H_2)] \quad (16)$$

$$J_0^0 = H_1 - H_2$$

$$G_n^m = \int_0^z sha_n^m(z-\xi)d\xi\,, \qquad G_0^0 = \int_0^z (z-\xi)d\xi \quad (17)$$

$$B_n^m = \gamma_n^m\cdot S_n^m(z) + \frac{g\cdot cha_n^m z}{\gamma_n^m\cdot cha_n^m h}, \qquad S_n^m(z) = \frac{sha_n^m z}{a_n\cdot cha_n h} \quad (18)$$

$$a_n^m = \sqrt{a_n^2 + a_m^2}, \qquad \gamma_n^m = \sqrt{a_n^m tha_n^m}$$

$$a_n = \frac{n\pi}{l}\,, \qquad a_m = \frac{m\pi}{l} \quad (19)$$

$$n = 1,2,\dots\,, \qquad m = 1,2,\dots$$

If we consider the hydrodynamic process after land sliding to the reservoir, i.e. when $t > t_0$; $f_1(t) = 0$ the expressions will be simplified and

$$\varphi_0^0 = -\psi_0^0 g\cdot i_0^0(t)\cdot J_0^0$$
$$\varphi_n^m = -\psi_n^m\cdot B_n^m(z)\cdot i_n^m(t)\cdot J_n^m \quad (20)$$
$$n,m = 0,1,2,\dots$$

In this case an amplitude η of wave generated in the reservoir, will be calculated according to the following formula

$$\eta(x,y,t) = -\frac{1}{4g}(\varphi_0^0)_t' - \frac{1}{2g}\sum_{m=1}^{\infty}(\varphi_0^m)_t' cosa_m y - \frac{1}{2g}\sum_{n=1}^{\infty}(\varphi_m^0)_t' cosa_n x - \sum_{n=1}^{\infty}\sum_{m=1}^{\infty}(\varphi_n^m)_t' cosa_m y\cdot cosa_n x \quad (21)$$

Where

$$(\varphi_n^m)_t' = \frac{\partial\varphi_n^m}{\partial t} \quad (22)$$

Or else in the alternate form:

$$\eta(x,y,t) = -\frac{1}{4}\eta_0^0 - \frac{1}{2}\sum_{n=1}^{\infty}\eta_n^0 cos a_n x - \frac{1}{2}\sum_{m=1}^{\infty}\eta_0^m cos a_m y + \sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\eta_n^m cos a_n x \cdot cos a_m y \quad (23)$$

Where

$$\eta_0^0 = \psi_0^0 J_0^0 [i_0^0(t)]_t', \qquad \eta_n^m = \frac{\psi_n^m \cdot J_n^m}{\gamma_n^m \cdot cha_n^m h}[i_n^m(t)]_t' \quad (24)$$

$$[i_n^m(t)]_t' = \frac{\partial i_n^m(t)}{\partial t} = 2sin\gamma_n^m \frac{t_0}{2} \cdot cos\gamma_n^m\left(t - \frac{t_0}{2}\right)$$
$$[\, i_0^0(t)]_t' = \frac{\partial j_0^0(t)}{\partial t} = t_0 \quad (25)$$

$$\psi_n^m = -\frac{4U_1}{ll_1 a_n}\begin{cases} x_2 - x_1 = 2b & ; n = m = 0 \\ sin a_n x_2 - sin a_n x_1 & ; n > 0, m = 0 \end{cases} \quad (26)$$

When the landslide (mud flow) mass or water inflows in the initial $x = 0$ cross-section, and the main flow is oriented along the x axis, then the wave processes generated in the reservoir will be described as follows:

$$\psi_n^m = -\frac{4V_1}{ll_1 a_m}\begin{cases} y_2 - y_1 = 2b_1 & ; n = m = 0 \\ sin a_m y_2 - sin a_m y_1 & ; n = 0, m > 0 \end{cases} \quad (27)$$

Where $2b_1$ and V_1 (or outflow$-V_1$) are the width (along the y axis) and velocity of landslide mass inflow section.

As is seen from solutions, the value of wave amplitude generated in the reservoir due to landslide varies according to x, y coordinates and t time. At the same time, it is a function of a wide variety of parameters, namely

$$\eta(x,y,t) = f(l, l_1, h, U_1, t_0, 2b, x_0, H_1, H_2) \quad (28)$$

As far as η value is in direct functional dependence from U_1 velocity, presuming that $H_1 = 0, H_2 = h$ we can represent the dependence as follows:

$$\frac{\eta(x,y,t)}{U_1} = f(l, l_1, h, t_0, 2b, x_0) \quad (29)$$

Let us enter the notion of parameter $D = U_1 t_0$, which expresses an average thickness of a landslide mass slided down to the reservoir. Based on the obtained computation pattern it can be assessed in a following way:

$$\frac{\eta(x,y,t)}{D} = \frac{1}{t_0} f(l, l_1, h, t_0, 2b, x_0) \quad (30)$$

While expressing the calculation parameters entering the expression in terms of relative magnitudes, we obtain

$$\frac{\eta^*(x^*, y^*, t^*)}{D^*} = \frac{1}{t_0^*} f(l^*, {l_1}^*, {t_0}^*, 2b^*, {x_0}^*) \quad (31)$$

Where

$$l^* = \frac{l}{h}, \quad {l_1}^* = \frac{l_1}{h}, \quad 2b^* = \frac{2b}{h}$$
$${x_0}^* = \frac{x_0}{h}, \quad {t_0}^* = t_0\sqrt{\frac{g}{h}}, \quad x^* = \frac{x}{h}$$
$$y^* = \frac{y}{h}, \quad t^* = t\sqrt{\frac{g}{h}}, \quad D^* = \frac{D}{h} \quad (32)$$
$$\eta^* = \frac{\eta}{h}$$

It is much more convenient to characterize the wave process generated via landslide impact through calculation of parameters written in the form of $\eta/_D$ dependence of amplitude from the average landslide thickness

Conclusion

The discussed methodology makes it possible to set up a computer program for mud flow forming wave calculation in the schematized water reservoir according to the different directions of mud flow movement that is very important at the stage of planning, construction and operation of reservoirs and dams.

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