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Issue **Article**

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THE EFFECT OF HETEROGENEITY OF VARIANCE DATA ON PARAMETRIC AND NONPARAMETRIC REGRESSION MODELS

***Abstract**: This study examined the effect of heterogeneity of variance data on parametric regression (OLS) and nonparametric regression (quantile regression- QR) models. The study was first subjected to heterogeneity of variance test via Breusch-Pagan-Godfrey technique, and it was revealed that there was existence of heterogeneity of variance in the data employed for the study. The multiple regression model of five explanatory variables, viz: shoulder width, elbow height, sitting height, arm length and age and the response variable (cholesterol) was first fitted with the adjusted coefficient of determination of 70.1% with the AIC being 26.245, as well as the quantile regression whose adjusted pseudo together with AIC are:* $(0.523, 13.745)$, $(0.375, 32.911)$, *and* $(0.558, 18.314)$ *for 25%, 50% and 75% respectively. The AIC agreed with the fact that the QR model was the best over the OLS model when there is presence of heterogeneity of variance in the data. The stepwise regression revealed that only three predictor variables (elbow height, age and shoulder width) were significantly related to the response variable at 5% level of significance. Comparison of parametric and nonparametric regression as the number of predictor variable increased to two and three also detected the presence of heterogeneity of variance, which gave QR advantage over OLS via their AIC values.*

***Key words**: Parametric Regression, Nonparametric Regression, heterogeneity of variance, Pseudo-Values, AIC.*

***Language**: English*

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Introduction

One of the main objectives of a parametric regression analysis in classical form is to set up an association between a dependent and explanatory variable. Regression analysis in classical form makes use of the mean and as well a function which suggests the conditional mean of the dependent value in respect of any constant predictor variable (Gürsakal et al, 2016).

Parametric regression analysis in classical form is problem free when its hypotheses are proven in an ideal condition. According to Jalali and Babanezhad (2011), these hypotheses are not always conformable to the real world, and as a result could lead to presence of outliers and heavy tailed distributions. An approach in general in regression analysis in classical form is to detect the outliers, and thereafter expunge them, which results in loosing valuable data points. Nonparametric regression also known as Quantile regression (QR) to be examined in this study, on the other hand gives a clearer comprehension of presence of outliers at the end of the tails of the distribution instead of expunging them. This condition proves effectiveness of the nonparametric quantile regression over the regression analysis in classical form (Giambona & Porcu, 2015). According to Koenker

(2005), QR is a robust approach of regression which neglects residual terms for the normal distribution, which is a more appropriate approach to handle such a condition.

According to Pan and Leu (2016), QR is best implemented in situations when the conditional quantiles reflect variations. In regression analysis, heterogeneity of variance is a situation where the variance error is not constant within all observations, where as it should be homoscedastic in agreement with one of the assumptions of OLS, this gives QR an advantage over OLS when there is presence of heterogeneity of variance in the data set (Draper & Smith, 1998).

2. Materials and Methods

2.1 Multiple Linear Regression Model

Assuming there are p variables for prediction of Y, the dependent variable (Faraway, 2002). The p explanatory variables are labeled $Z_1, Z_2, \cdots, Z_p$, the stages of these variables for the ith case is denoted Z_{1i} ,..., Z_{pi} .

$$E(Y/Z) = \phi_o + \phi_1 Z_1 + \phi_2 Z_2 + \ldots + \phi_p Z_p + \varepsilon \quad (1)$$

2.2 ANOVA Table for Regression Analysis

Table 1: Analysis of variance (based on p predictor variables)

Source	df	SS	MS	F
Regression	df_R	SS_R	$MS_R = SS_R / p$	$F_{obs} = MS_R / MS_E$
Error	df_E	SS_E	$MS_E = SS_E / (n-p-1)$	
Total	df_T	SS_T	...	...

Sum of squares for total and df: $SS_T = \sum_{i-1}^{n} (Y_i - \bar{Y})^2 \qquad df_T = n-1$

Sum of squares for regression: $SS_R = \sum_{i-1}^{n} (\hat{Y}_i - \bar{Y})^2 \qquad df_R = p$

Sum of squares for error: $SS_E = \sum_{i-1}^{n} (Y_i - \hat{Y})^2 \qquad df_E = n-p-1$

2.3 Multiple Coefficient of Determination(R^2)

The R^2 is given as;

$$R^2 = \frac{\sum_{i=1}^{n} (\hat{Y}_i - \bar{Y})^2}{\sum_{i=1}^{n} (Y_i - \bar{Y})^2} = \frac{SS_R}{SS_T} = 1 - \frac{SS_E}{SS_T} \qquad (2)$$

The R^2 adjusted represented as, R_a^2 is defined by

$$R_a^2 = 1 - \left(\frac{n-1}{n-p}\right) \frac{SS_E}{SS_T} \qquad (3)$$

2.4 Quantile Regression

The general quantile regression model according to Buchinsky (1998) is as follows:

$$y_i = z'\phi_\lambda + \varepsilon_{\lambda i}, i = 1,2,..., n \qquad (4)$$

Where:

y_i denotes the response variable and the λth quantile $(0 < \lambda < 1)$ of the conditional distribution of y_i is a linear function of a $p \times 1$ vector of explanatory variables, z_i and an unknown error term $\varepsilon_{\lambda i}$; ϕ_λ is the unknown parameters of regression in vector form which is connected with percentiles. The conditional quantile function can be expressed as $Q_\lambda\left(y_i / z_i\right) = z'\phi$. The quantile regression minimizes a sum which produces the asymmetric penalties $\lambda|\varepsilon|$ for under-prediction and $(1-\lambda)|\varepsilon|$ for over-prediction. The quantile regression estimator $\hat{\phi}_\lambda$ minimizes over ϕ the objective function

$$Q(\phi_\lambda) = \min_{\phi_\lambda} \left\{ \sum_{y_i \geq z'\phi_\lambda} \lambda |y_i - z'\phi_\lambda| + \sum_{y_i \leq z'\phi_\lambda} (1-\lambda) |y_i - z'\phi_\lambda| \right\} \qquad (5)$$

The study considers three quantile regressions at the 25th, 50th and 75th quantiles; where λ is called the regression quantile, $0 < \lambda < 1$.

2.5 Computing QR using Two -Variables Problem

The two-variable problem for L_1 criterion for minimization is given by:

$$\sum_{i=1}^{n} \left|\phi_0 + \phi_1 z_i - y_i\right| \quad (6)$$

The resulting linear program is:

$$\left.\begin{aligned} &\min imize \sum_{i=1}^{n} \varepsilon_i \\ &Subject\,to: \varepsilon_i \geq \phi_0 + \phi_1 z_1 - y_i,\ i = 1,2,\cdots,n \\ &\qquad\qquad \varepsilon_i \geq -(\phi_0 + \phi_1 z_1 - y_i),\ i = 1,2,\cdots,n \end{aligned}\right\} \quad (7)$$

Each ε_i is an auxiliary variable. The constraints guarantee that:

$$\varepsilon_i \geq \max\{\phi_0 + \phi_1 z_i - y_i, -(\phi_0 + \phi_1 z_1 - y_i)\} = \left|\phi_0 + \phi_1 z_i - y_i\right| \quad (8)$$

To evaluate the L_1 regression case, it requires to evaluate the equivalent linear programming problem (Gürsakal et al, 2016).

$$Q_\lambda(\hat{y}/z) = \hat{\phi}_0(\lambda) + \hat{\phi}_1(\lambda) z \quad (9)$$

The error absolute sum of weighted differences is the associated minimizer

2.6 QR Goodness of Fit

In the simplest form of regression equation with one independent variable

$$EASW_\lambda = \sum_{y_i \geq V} \lambda\left|y_i - \hat{\phi}_0(\lambda) - \hat{\phi}_1(\lambda) z_i\right| + \sum_{y_i \geq V} (1-\lambda)\left|y_i - \hat{\phi}_0(\lambda) - \hat{\phi}_1(\lambda) z_i\right| \quad (10)$$

Where $V = \hat{\phi}_0(\lambda) + \hat{\phi}_1(\lambda) z_i$

The total absolute sum of weighted differences between the observed dependent variable and the estimated quantile according to Koenker & Machado (1999) is by:

$$TASW_\lambda = \sum_{y_i \geq \lambda} \lambda\left|y_i - \hat{\lambda}\right| + \sum_{y_i < \lambda} (1-\lambda)\left|y_i - \hat{\lambda}\right| \quad (11)$$

The obtained *pseudo* R^2 is evaluated using:

$$pseudo\ R_\lambda^2 = 1 - \frac{EASW_\lambda}{TASW_\lambda} \quad (12)$$

$EASW_\lambda < TASW_\lambda$, the $pseudo\ R_\theta^2$ runs from 0 to 1(Hao & Naiman, 2013).

1. **Results**

3.1 Testing for Heterogeneity of Variance

The hypotheses of the Breusch-Pagan-Godfrey test are as follows:

Table 3: Test for Heterogeneity of Variance

F-stat.	12.189	Prob. F(5,44)	0.000
Obs*R^2	29.037	Prob. Chi-Square(5)	0.000

Table 3 shows that heterogeneity of variance does seem to be a problem since the p-value (0.000) is lower than 0.05. Hence, the null hypothesis is rejected in testing for heterogeneity of variance.

3.2 QR and OLS Regression

Table 4: QR Model

	QR					
	25%		50%		75%	
Parameter	Coef.	Prob.	Coef.	Prob.	Coef.	Prob.
Constant	171.037	0.000	169.415	0.000	169.859	0.000
Shoulder Width	-4.524	0.004	-3.676	0.037	-0.072	0.956
Elbow Height	2.995	0.001	3.839	0.000	2.708	0.002
Sitting Height	0.384	0.464	0.302	0.261	0.127	0.622
Arm Length	-0.007	0.995	0.819	0.459	-0.068	0.946
Age	-0.020	0.068	-0.023	0.004	-0.019	0.006
	$R_{\lambda}^2 = 0.572$		$R_{\lambda}^2 = 0.439$		$R_{\lambda}^2 = 0.603$	
	$Adj.\ R_{\lambda}^2 = 0.523$		$Adj.\ R_{\lambda}^2 = 0.375$		$Adj.\ R_{\lambda}^2 = 0.558$	
	$AIC = 13.745$		$AIC = 32.911$		$AIC = 18.314$	

Table 5: OLS Regression Model

Parameter	Coef.	Std. Error	t-stat	Prob.
Constant	169.473	1.383	122.541	0.000
Shoulder Width	-1.978	0.857	-2.309	0.026
Elbow Height	3.188	0.484	6.581	0.000
Sitting Height	0.272	0.214	1.268	0.212
Arm Length	0.234	0.826	0.283	0.778
Age	-0.016	0.004	-3.459	0.001
$R^2 = 0.731\ Adj.\ R^2 = 0.701$				
$AIC = 26.245$				

Table 4 reveals the outputs of fitting a quantile regression model to explain the association between cholesterol and five anthropometric measurements. The Akaike Information Criterion (AIC) for QR (25%), QR (50%) and QR (75%) are 13.745, 32.911 and 18.314 respectively. The fitted model is

$$Cholesterol = 171.037 - 4.524\ Shoulder\ width + 2.995\ Elbow\ height + 0.384\ Sitting\ height - 0.007\ Arm\ length - 0.020 Age$$

Table 5 reveals the outputs of fitting a regression model with multiple predictor variables to explain the association between cholesterol and five anthropometric measurements. The fitted model is

$$Cholesterol = 169.473 - 1.978\ Shoulder\ width + 3.188\ Elbow\ height + 0.272\ Sitting\ height + 0.234\ Arm\ length - 0.016 Age$$

It has been noticed that the largest p-value on the predictor variables is 0.778 in examining the simplification of the model which belongs to arm length, and since it is greater than 0.05, then arm length is not statistically significant at 95.0% or higher confidence level. We consider expunging it from the model and conduct a stepwise regression. The Akaike Information Criterion (AIC) is 26.245. Considering the values of AIC, it was observed that the quantile regression model is more adequate to explain the association between the response variable and the five independent variables.

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Table 6: Stepwise Regression

Parameter	Coef.	SE	t-stat	Prob
Constant	170.515	0.801	212.871	0.000
Elbow Height	2.818	0.374	7.529	0.000
Age	-0.017	0.004	-3.899	0.000
Shoulder Width	-2.088	0.838	-2.493	0.016

Table 6 displays the summary of the output for a stepwise regression model for the association between cholesterol and the anthropometric measurements. The fitted model is

$$Cholesterol = 170.515 + 2.818 \text{ Elbow Height} - 0.017\ Age - 2.088\ ShoulderWidth$$

Since the p-values of all the predictor values are less than 0.05, it indicates significant association statistically between the variables at 5% level of significance. The coefficient of determination value shows that the fitted model explains 72.2% of the variability in cholesterol, while the adjusted coefficient of determination value, which is more appropriate for comparing models with different number of predictor variables is 70.3%. Hence, there is no need to further simplify the model since the largest p-value amongst the predictor variable is 0.016 corresponding to Shoulder Width, which is significant. There is no need to expunge any variable from the model.

3.3 Comparison of OLS and QR with Increase in Variables

Table 7: Numerical Result of the OLS and QR Simple Linear Models

			QR					
	OLS		25%		50%		75%	
Parameter	Coeff.	Prob.	Coeff.	Prob.	Coeff.	Prob.	Coeff	Prob
Constant	167.862	0.000	168.250	0.000	167.303	0.000	168.595	0.000
Elbow Height	3.440	0.000	2.917	0.001	3.871	0.000	3.095	0.000
	$R^2 = 0.602$		$R_\lambda^2 = 0.389$		$R_\lambda^2 = 0.300$		$R_\lambda^2 = 0.404$	
	$Adj.\ R^2 = 0.593$		$Adj.\ R_\lambda^2 = 0.377$		$Adj.\ R_\lambda^2 = 0.286$		$Adj.\ R_\lambda^2 = 0.392$	

Table 7 shows the summary output of fitting the OLS regression in line with that of quantile regression. The estimated coefficient for the elbow height reveals a positive and significant relationship on cholesterol as a measure of health. However, the normality test for the residual using the Anderson-Darling statistic (0.026) shows that it is not normally distributed. This definitely leads to OLS estimates being inappropriate to employ, thereby giving the quantile regression a great advantage since quantile regression does not assume normally distributed errors for the estimation of the coefficients, whereas OLS does for simple linear regression.

Table 8: Numerical Result of the OLS and QR Models with Two Predictor Variables

			QR					
	OLS		25%		50%		75%	
Parameter	Estimate	p-value	Estimate	p-value	Estimate	p-value	Estimate	p-value
Constant	169.047	0.000	168.697	0.000	168.723	0.000	170.631	0.000
Elbow Height	3.094	0.000	2.955	0.001	3.368	0.001	2.222	0.001
Age	-0.016	0.001	-0.011	0.440	-0.016	0.035	-0.021	0.000
	$R^2 = 0.684$		$R_\lambda^2 = 0.406$		$R_\lambda^2 = 0.377$		$R_\lambda^2 = 0.588$	

	$Adj.\ R^2 = 0.670$	$Adj.\ R_\lambda^2 = 0.381$	$Adj.\ R_\lambda^2 = 0.351$	$Adj.\ R_\lambda^2 = 0.571$
	$AIC = 28.376$	$AIC = 40.487$	$AIC = 37.348$	$AIC = 15.887$

Table 8 shows the summary output of fitting the multiple linear regression in line with that of quantile regression. The estimated coefficient for the elbow height reveals a positive and significant relationship on cholesterol as a measure of health, while age has significant negative effect on cholesterol. However, Breusch-Pagan-Godfrey for heterogeneity of variance shows that heterogeneity of variance is present in the data with a p-value of 0.001. This definitely leads to OLS estimates being inappropriate to employ, thereby giving the quantile regression a great advantage since quantile regression does not assume homoscedasticity presence to be achieved. Quantile regression improves the efficiency of the estimators compared to OLS and allows analyzing it independently. Considering the values of AIC, it also proved that the quantile regression model is more appropriate to explain the association between the dependent variable and the five independent variables.

Table 9: Numerical Result of the OLS and QR Models with Three Predictor Variables

			QR					
	OLS		25%		50%		75%	
Parameter	Coeff.	Prob.	Coeff.	Prob.	Coeff.	Prob.	Coeff.	Prob.
Constant	170.515	0.000	171.010	0.000	172.065	0.000	170.631	0.000
Elbow Height	2.818	0.000	2.850	0.000	2.920	0.000	2.222	0.000
Age	-0.017	0.000	-0.020	0.252	-0.031	0.003	-0.021	0.001
Shoulder Width	-2.088	0.016	-3.435	0.065	-4.188	0.024	-0.000	1.000
	$R^2 = 0.722$		$R_\lambda^2 = 0.524$		$R_\lambda^2 = 0.419$		$R_\lambda^2 = 0.588$	
	$Adj.\ R^2 = 0.703$		$Adj.\ R_\lambda^2 = 0.493$		$Adj.\ R_\lambda^2 = 0.381$		$Adj.\ R_\lambda^2 = 0.562$	
	$AIC = 24.039$		$AIC = 20.420$		$AIC = 32.466$		$AIC = 17.887$	

Table 9 shows the summary output of fitting the multiple linear regression in line with that of quantile regression. The estimated coefficient for the elbow height reveals a positive and significant relationship on cholesterol as a measure of health, while age and shoulder width have significant negative effect on cholesterol. However, Breusch-Pagan-Godfrey for heterogeneity of variance shows that heterogeneity of variance is present in the data with a p-value of 0.000. This definitely leads to OLS estimates being inappropriate to employ, thereby giving the quantile regression a great advantage since quantile regression does not assume homoscedasticity presence to be achieved. Quantile regression improves the efficiency of the estimators compared to OLS and allows analyzing it independently. Considering the values of AIC statistic, it also proved that the quantile regression model is more appropriate to describe the relationship between the dependent variable and the five predictor variables.

4. Discussion

The data set of the study was first subjected to heterogeneity of variance test via the Breusch-Pagan-Godfrey statistic and the result revealed that heterogeneity of variance was present in the data since the p-value is 0.000. The model for both the OLS and QR were fitted using the result of Tables 4 and 5. In determining whether the model of the OLS could be simplified, it was noticed that the largest p-value on the predictor variables is 0.778 which belongs to arm length, and since it is greater than 0.05, then arm length is not statistically significant at 95.0% or higher confidence level. We considered expunging it from the model and conduct a stepwise regression. Again, the Akaike Information Criterion (AIC) for OLS is 26.245. Considering the values of AIC, it was observed that the quantile regression model is more appropriate to explain the association between the response variable and five predictor variables. The OLS and QR were compared as the number of variable increases to two and three and it was discovered the there were presence of heterogeneity of variance via Breusch-Pagan-Godfrey; hence; giving the quantile regression a great advantage since quantile regression does not assume homoscedasticity presence to be achieved. The Akaike Information Criterion (AIC) result also proved the QR model is appropriate over the OLS model.

5. Conclusion

The study concluded that non-parametric quantile regression is better employed especially when there is presence of non-constant residual terms which violated the major assumption of OLS (homoscedasticity).

6. Recommendations

Based on publicly available dataset on cholesterol and anthropometric dimensions of patients, quantile regression model outperforms the OLS method based on the AIC values. It is recommended that future researchers should still employ the nonparametric quantile regression when important assumptions associated with OLS hold. Again, since the data set employed in this study is a real life situation, simulation of data of different sample sizes (violated and non violation of assumptions) should be looked at by future researchers.

References:

1. Draper, N. R., & Smith, H. (1998). *Applied regression analysis* (Vol. 326). John Wiley & Sons.
2. Faraway, J. J. (2002). *Practical regression and ANOVA using R* (Vol. 168). Bath: University of Bath.
3. Giambona, F., & Porcu, M. (2015). Student background determinants of reading achievement in Italy. A quantile regression analysis. *International Journal of Educational Development*, *44*, 95-107.
4. Gürsakal, S., Murat, D., & Gürsakal, N. (2016). Assessment of PISA 2012 results with quantile regression analysis within the context of inequality in educational opportunity. *Alphanumeric Journal*, *4*(2), 41-54.
5. Gürsakal, S., Murat, D., & Gürsakal, N. (2016). Assessment of PISA 2012 results with quantile regression analysis within the context of inequality in educational opportunity. *Alphanumeric Journal*, *4*(2), 41-54.
6. Jalali, N., & Babanezhad, M. (2011). Quantile regression due to skewness and outliers. *Applied Mathematical Sciences*, *5*(39), 1947-1951.
7. Koenker, R. (2005). *Quantile regression* (Econometric Society Monographs). Cambridge: Cambridge University Press.
8. Koenker, R., & Machado, J. A. (1999). Goodness of fit and related inference processes for quantile regression. *Journal of the American statistical association*, *94*(448), 1296-1310.
9. Pan, W. T., & Leu, Y. (2016). An analysis of bank service satisfaction based on quantile regression and grey relational analysis. *Mathematical Problems in Engineering*.