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Issue Article

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A MATHEMATICAL MODEL OF A TECHNICAL SYSTEM ELEMENT

Abstract*: A mathematical model of an element of a technical system has been obtained using the principled approach. The element of the technical system includes a thermistor. Conductivity of the thermistor depends on its temperature in the temperature range under consideration. The built mathematical model possesses the properties of fullness, adequacy, productivity and economy to a sufficient degree. The use of such a model reduces the time and money spent on conducting research and enables the rational use of mathematical modeling capabilities.*

Key words*: thermistor, mathematical model, properties of mathematical models, principled approach.*

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Introduction

Approaches to building mathematical models of various technical systems are described in the extensive training and research literature. Basic principles of operation of thermistors have been studied, their performance data have been established, methods for designing circuits with thermistors have been considered, and numerous examples of their successful practical use in various fields of human activity are known so far.

The paper [1] clarifies the concept of "mathematical modeling", which is defined as the replacement of the object of study with a useful mathematical model and its subsequent study by known methods, and the mathematical model is deemed useful if it has the necessary properties in relation to a particular study to a sufficient degree. Some properties of mathematical models are given e.g. in [1; 2]. Combination of said properties determines the requirements for the mathematical model. Such requirements are contradictory and in practice can be satisfied on the basis of a reasonable compromise, for which the rules are usually observed and the recommendations obtained as a result of generalization of practical experience gained in the building of mathematical models are usually followed (see e.g. [2–5]). The paper [6] discusses an example of building a mathematical model that possesses the necessary properties to a sufficient degree in relation to the specific study, and some of the results thereof are detailed in [7; 8]. Features of introduction of the principled approach to building mathematical models are considered in [9–12].

The purpose of this work is to develop a useful mathematical model of an element of a technical system using the principled approach. The element of the technical system includes a thermistor, conductivity of which depends on its temperature.

Problem statement

Let us assume T is the temperature of the thermistor, which does not depend on the spatial coordinates. Temperature T at the initial instant of time t_0 is equal to T_0. Total heat capacity of the thermistor is equal to the constant value C. Convective heat exchange with the environment occurs on the thermistor surface with area S. The ambient temperature is equal to T_0, and the heat transfer coefficient is known and equal to α. For the temperature range from T_0 to T^*, it is assumed that the conductivity of the thermistor is equal to

$$G(T) = g\left[1+\beta(T-T_0)\right]^{-1},$$

where g is the conductivity of the thermistor at $T = T_0$; β is the thermal coefficient, where $\beta > 0$. Electric current flows through the thermistor, and its intensity is

$$I = gU\left[1+\beta(T-T_0)\right]^{-1}, \qquad (1)$$

where U is the constant electrical potential difference at the poles of the element under consideration.

Let I be the value of interest in the study. A mathematical model of the object of study, which possesses the properties of fullness, adequacy, productivity and economy to a sufficient degree is to be built.

Solution

According to the principle of gradual complication, in order to solve the problem, a hierarchy of mathematical models of the given object of study is built and the conditions under which it is possible to find the desired value I with a relative error of no more than the given value δ_0 are determined.

If the difference $T - T_0$ is small enough, then according to (1), the desired value is obtained using the formula

$$I_0 = gU. \qquad (2)$$

Conditions under which the obtained formula is applicable are to be determined. A steady-state heat exchange process is considered for this reason. In this case, the heat generation power in the thermistor material is equal to the heat flow removed from the thermistor, i.e.

$$UI_0\left[1+\beta(T_*-T_0)\right]^{-1} = \alpha(T_*-T_0)S,$$

where T_* is the steady-state temperature value of the thermistor, where $T_* \le T^*$. From the inequation obtained, it is easy to find

$$T_* = T_0 + \frac{1}{2\beta}\left(-1+\sqrt{1+4\beta UI_0/(\alpha S)}\right),$$

and then determine the steady-state value

$$I_* = G(T_*)U = \frac{2I_0}{1+\sqrt{1+4\beta UI_0/(\alpha S)}}. \qquad (3)$$

It is obvious that $I_* \le I \le I_0$. Then, the relative error of the value I_0 is

$$\delta(I_0) = \left|\frac{I-I_0}{I}\right| = \frac{I_0}{I} - 1 \le \frac{I_0}{I_*} - 1.$$

Consequently, with the following condition is satisfied

$$\frac{I_0}{I_*} - 1 \le \delta_0$$

formula (2) can be used with a relative error of no more than δ_0 to find the desired value. Then, the following inequation is obtained

$$\beta UI_0/(\alpha S) \le (\delta_0+1)\delta_0. \qquad (4)$$

Subject to this inequation satisfied, the mathematical model (2) possesses the properties of fullness, adequacy, productivity and economy to a sufficient degree.

Conditions are to be determined under which mathematical model (3) is applicable. The unsteady-state heat exchange process is considered for this reason. In this case, the change of the thermistor temperature in time t describes the first-order ordinary differential equation

$$C\frac{dT}{dt} = UI_0\left[1+\beta(T-T_0)\right]^{-1} - \alpha(T-T_0)S,$$

and the initial condition is

$$T(t_0) = T_0.$$

Considering that

$$I = I_0\left[1+\beta(T-T_0)\right]^{-1},$$

the Cauchy problem is defined

$$CI_0\frac{dI}{dt} = \alpha S(I_0 - I)I - \beta UI^3, \qquad (5)$$
$$I(t_0) = I_0.$$

With the following condition satisfied

$$\delta(I_*) = \left|\frac{I-I_*}{I}\right| = 1 - \frac{I_*}{I} \le \delta_0$$

formula (3) can be used with a relative error of no more than δ_0 to find the desired value, where

$$\delta_0 < \frac{I_0}{I_*} - 1,$$

since formula (2) should be used otherwise. Then, the following instant of time is obtained

$$t_* = t_0 + \frac{C}{\alpha S}\left[\frac{I_*-I_0}{2I_0-I_*}\ln\left(2-\frac{I_*}{I_0}-\delta_0\right) - \frac{I_0}{2I_0-I_*}\ln\left(\frac{I_0}{I_0-I_*}\delta_0\right)\right],$$

for which $I(t_*) = I_*/(1-\delta_0)$. Then according to (5), the steady-state value I_* can be considered to be equal to $I(t)$ at $t \ge t_*$ with a relative error not exceeding δ_0.

If condition (4) is not met, then mathematical model (3) at $t \ge t_*$ possesses the properties of fullness, adequacy, productivity and economy to a sufficient degree.

The development of a new mathematical model in the event of the formation of a hierarchy of mathematical models of the object of study might lead to the clarification of the previously determined conditions for the applicability of the built mathematical models. Indeed, it is possible to clarify the condition of applicability of formula (2) using the mathematical model (5). The instant of time is to be determined for this reason

$$t^* = t_0 + \frac{C}{\alpha S}\left[\frac{I_* - I_0}{2I_0 - I_*}\ln\left(1 + \frac{I_*}{I_0}\delta_0\right) - \right.$$
$$\left. - \frac{I_0}{2I_0 - I_*}\ln\left(1 - \frac{I_*}{I_0 - I_*}\delta_0\right)\right],$$

for which $I(t^*) = I_0/(1+\delta_0)$. Then, the value I_0 can be considered equal to $I(t)$ at $t_0 \le t \le t^*$ with a relative error of no more than δ_0.

If condition (4) or $t \le t^*$ is met, then the mathematical model (2) possesses the properties of fullness, adequacy, productivity and economy to a sufficient degree.

Results

For the temperature range from T_0 to T_*, the following statements are valid, which reveal a useful mathematical model of the object of study.

Statement 1. If condition (4) or $t \le t^*$ is satisfied, then the mathematical model (2) is deemed useful.

Statement 2. If condition (4) is not satisfied, then the mathematical model (3) is deemed useful at $t \ge t_*$.

Statement 3. If condition (4) does not hold, while the time interval from t^* to t_* is of interest, then the mathematical model (5) is considered as useful.

Conclusion

Thus, following the approach used, the statements are defined that allow to establish a useful mathematical model of the element of the technical system. The built mathematical model possesses the properties of fullness, adequacy, productivity and economy to a sufficient degree with respect to this study.

The use of such a model obviously reduces the time and money spent on conducting research and enables the rational use of mathematical modeling capabilities.

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