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SECTION 7. Mechanics and machine construction.

Mukatay Nemerebayev
doctor of technical Sciences, Professor,
Taraz innovation and humanities University, Kazakhstan
nemerebayev@mail.ru

Murat Musabekovich Bekmuratov
Candidate of technical sciences, associate Professor,
Taraz state University named after M.Kh. Dulati,
Kazakhstan

Seitzhan Orynbayev
PhD, associate Professor,
Taraz state University named after M.Kh. Dulati,
Kazakhstan
seitzhan_74@mail.ru

Aigerim Nemerebayeva
Master of Laws,
Taraz innovation and humanities University, Kazakhstan

EFFECT OF SHEAR DEFORMATION ON FREE VIBRATION OF CYLINDRICAL COMPOSITE NETLIKE SHELLS

Abstract: This paper describes the effect of shear deformation on free vibration of composite netlike cylindrical shell.

Using "additional" forces N_{ii}, N_{ij} at critical deviation equilibrium of shell in the linear equation, derived from technical theory, and displacement fields u, v, and w, defining the relative deformation, derived the limited equation of free vibration of netlike cylindrical shells.

Also based on proven theory of S.A.Ambartsumyan and using displacement fields u, v, and w derived the equation for free vibration frequency of netlike cylindrical shell with the effect of shear deformation.

The relative difference $\delta(m,n)$ of the two vibration frequency equations at constant *n=3, n=10* was plotted against increasing m and it was proven that the difference increases with the effect of shear deformation.

Key words: *vibration, deformation, frequency, shell.*
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1. Introduction.

One of the problems of used shell theories based on Kirchoff-Love kinematical model is that it does not account for shear deformation. This factor effects determining the vibration frequency and critical static force, thus there is no need to prove that it will have its impact when solving complex dynamic problems. This paper describes the use of technical theory based on Kirchoff-Love kinematical model on composite cylindrical netlike shells.

Nowadays when solving problems of plates and shells taking into account shear deformation, the linear theory of cross-sectional tangent deflections of structures (The Timoshenko kinematic model) and the theory based on parabolic shear stress on cross-section are mostly used [1, 2, 3].

Analyzing the papers describing the determination the frequency of free vibration using the shear deflection and rotatory inertia of isotropic cylindrical shells Greenberg et al [4] points on a problem of determining the free vibration frequency of anisotropic shells especially the effect of shear deformation on shell with low shear module.

2. The method to account for the effect of shear deformation on free vibration of composite netlike cylindrical shell.

From the technical theory we can derive the following linear equations for netlike cylindrical shell.

$$\frac{\partial N_{11}}{\partial x} + \frac{\partial N_{21}}{\partial y} + P_1 + X_1 = 0,$$

$$\frac{\partial N_{22}}{\partial y}+\frac{\partial N_{12}}{\partial x}+P_2+X_2=0,$$

$$\frac{\partial N_{13}}{\partial x}+\frac{\partial N_{23}}{\partial y}-\frac{N_{22}}{R}+P_3+X_3=0, \quad (1)$$

$$\frac{\partial M_H}{\partial x}+\frac{\partial M_{21}}{\partial y}-N_{13}+\Phi_2+F_2=0,$$

$$\frac{\partial M_{22}}{\partial y}+\frac{\partial M_{12}}{\partial x}-N_{23}+\varphi_1+F_1=0,$$

here: N_{ij}, are the "additional" forces at critical deviation equilibrium of shell, and under the technical theory will take the following values.

$$N_{11}=C_{11}\varepsilon_1+C_{12}\varepsilon_2,$$
$$N_{22}=C_{22}\varepsilon_2+C_{12}\varepsilon_1,$$
$$N_{12}=N_{21}=C_{66}\varepsilon_{12},$$
$$N_{13}=C_{13}(\theta_1+\gamma_2), \quad (2)$$
$$N_{23}=C_{23}(\theta_2+\gamma_y),$$
$$M_{11}=D_{11}x_1+D_{12}x_2,$$
$$M_{22}=D_{22}x_2+D_{12}x_1,$$
$$M_{12}=M_{21}=D_{66}(\tau_1+\tau_2).$$

The relative deformations in the equation with the help of displacement fields *u*,*v* and *w* are defined as follows:

$$\varepsilon_1=\frac{\partial u}{\partial x};\varepsilon_2=\frac{\partial v}{\partial y}+\frac{w}{R},$$
$$\gamma_1=\frac{\partial v}{\partial x};\gamma_2=\frac{\partial u}{\partial y}, \quad (3)$$
$$\theta_1=\frac{\partial w}{\partial x};\theta_2=\frac{\partial w}{\partial y}.$$

$$x_1=\frac{\partial\gamma_x}{\partial x};x_2=\frac{\partial\gamma_y}{\partial y};\tau_1=\frac{\partial\gamma_y}{\partial x};\tau_2=\frac{\partial\gamma_x}{\partial y}. \quad (4)$$

Putting equations (2), (3) and (4) into equation (1) we will get the equation for free vibration of the shell.

$$C_{11}=\frac{\partial^2 u}{\partial x^2}+C_{66}\frac{\partial^2 u}{\partial y^2}+(C_{12}+C_{66})\frac{\partial^2 v}{\partial x\partial y}+\frac{c_{12}}{R}\frac{\partial\omega}{\partial x}=\frac{2\rho h\delta}{a}\left(\frac{\partial^2 u}{\partial t}+\frac{h^2\delta^2}{3a^2R}\frac{\partial^2\gamma_x}{\partial t^2}\right);$$

$$(C_{12}+C_{66})\frac{\partial^2 u}{\partial x\partial y}+C_{22}\frac{\partial^2 v}{\partial y^2}+C_{66}\frac{\partial^2 v}{\partial x^2}+\frac{C_{22}}{R}\frac{\partial w}{\partial y}=\frac{2\rho h\delta}{a}\left(\frac{\partial^2 u}{\partial t}+\frac{h^2\delta^2}{3a^2R}\frac{\partial^2\gamma_x}{\partial t^2}\right);$$

$$C_{13}\frac{\partial^2\omega}{\partial x^2}+C_{23}\frac{\partial^2\omega}{\partial y^2}-\frac{C_{22}}{R}\omega-\frac{C_{12}}{R}\frac{\partial u}{\partial x}-\frac{C_{22}}{R}\frac{\partial\vartheta}{\partial y}+C_{13}\frac{\partial\gamma x}{\partial x}+C_{23}\frac{\partial\gamma_y}{\partial y}=\frac{2\rho h\delta}{a}\frac{\partial^2\omega}{\partial t^2}; \quad (5)$$

$$D_{11}\frac{\partial^2\gamma_x}{\partial x^2}+D_{66}\frac{\partial^2\gamma_x}{\partial y^2}-C_{13}\gamma_x+(D_{12}+D_{66})\frac{\partial^2\gamma_y}{\partial x\partial y}-C_{13}\frac{\partial\omega}{\partial x}=\frac{2\rho h^3\delta^3}{3a^3}\left(\frac{\partial^2\gamma_x}{\partial t^2}+\frac{1}{R}\frac{\partial^2 u}{\partial t^2}\right);$$

$$D_{22}\frac{\partial^2\gamma_y}{\partial y^2}+D_{66}\frac{\partial^2\gamma_y}{\partial x^2}-C_{23}\gamma_y(D_{12}+D_{66})\frac{\partial^2\gamma_x}{\partial x\partial y}-C_{33}\frac{\partial\omega}{\partial y}=\frac{2\rho h^3\delta^3}{3a^3}\left(\frac{\partial^2\gamma_y}{\partial t^2}+\frac{1}{R}\frac{\partial^2\vartheta}{\partial t^2}\right).$$

Assuming that the shell is in free connection with two sides, and at $x=0$ values it is satisfied with finite conditions $N=0$ $\vartheta=0$ $\omega=0$ $M_x=0$, and putting the unknowns u,ϑ,ω and γ_x,γ_y into Fourier series we get:

$$U(x,y,t)=\sum_{m=1}^{\wp}\sum_{n=1}^{\wp}\left[U_{mn}(t)\cos\beta_n y+U'_{mn}(t)\sin\beta_n y\right]\cos\alpha_m x;$$

$$\vartheta(x,y,t)=\sum_{m=1}^{\wp}\sum_{n=1}^{\wp}\left[V_{mn}(t)\sin\beta_n y-V'_{mn}(t)\cos\beta_n y\right]\sin\alpha_m x;$$

$$\omega(x,y,t)=\sum_{m=1}^{\wp}\sum_{n=1}^{\wp}\left[W_{mn}(t)\cos\beta_n y-W'_{mn}(t)\sin\beta_n y\right]\sin\alpha_m x; \quad (6)$$

$$\gamma_x(x,y,t)=\sum_{m=1}^{\wp}\sum_{n=1}^{\wp}\left[x_{mn}(t)\cos\beta_n y + x'_{mn}(t)\sin\beta_n y\right]\cos\alpha_m x\,;$$

$$\gamma_y(x,y,t)=\sum_{m=1}^{\wp}\sum_{n=1}^{\wp}\left[Y_{mn}(t)\sin\beta_n y - Y'_{mn}(t)\cos\beta_n y\right]\sin\alpha_m x\,.$$

here $\alpha_m = \frac{\pi m}{L}, \beta_n = \frac{n}{R}$.

Putting equation (6) into equation (5) and by using two similar U_{mn}, V_{mn}, W_{mn}, X_{mn}, Y_{mn} and U'_{mn}, V'_{mn}, W'_{mn}, X'_{mn}, Y'_{mn} unknowns we get the series of equations, we write the first five differential equations in matrix form.

$$M\frac{d^2 f_{mn}}{dt^2} + G_{mn} f_{mn} = 0. \qquad (7)$$

Here $f_{mn} = \{U_{mn}, V_{mn}, W_{mn}, X_{mn}, Y_{mn}\}$ and G_{mn}, $\tilde{M}$ are next symmetric matrixes not equal to zero:

$$g_{mn}^{11} = \alpha_m^2 C_n + \beta_n^2 C_{66};$$

$$g_{mn}^{2} = -\alpha_m \beta_n (C_{12} + C_{66})$$

$$g_{mn}^{13} = -\frac{C_{12}}{R} dm;$$

$$g_{mn}^{22} = \alpha_m^2 C_{66} + \beta_n^2 C_{22};$$

$$g_{mn}^{23} = \frac{C_{22}}{R}\beta_n;$$

$$g_{mn}^{33} = \frac{C_{22}}{R^2 m} + C_{13}\alpha_m^2 + C_{23}\beta_n^2;$$

$$g_{mn}^{34} = C_{13}\alpha_m;$$

$$g_{mn}^{35} = -C_{23}\beta_n;$$

$$g_{mn}^{44} = \alpha_m^2 D_{11} + \beta_n^2 D_{66} + C_{13};$$

$$g_{mn}^{45} = -\alpha_m \beta_n (D_{12} + D_{66});$$

$$m_{11} = m_{22} = m_{33} = \frac{2\rho h\delta}{a};$$

$$m_{44} = m_{55} = \frac{2\rho h^3\delta^3}{3a^3};$$

$$m_{14} = m_{25} = \frac{2\rho h^3\delta^3}{3a^3 R}.$$

Analizing the effect of shear deformation on vibration frequency at bending vibration conditions in literature, and using equation (5) we get the known equation

$$\frac{d^2 W_{mn}}{dt^2} + \omega_{mn}^2 W_{mn} = 0. \qquad (8)$$

Here ω^2 value as per [5] determined as follows:

$$\omega_{mn}^2 = \frac{a}{2\rho h\delta}\left[\frac{\alpha_m^4 C_{66}(C_{11}C_{22} - C_{12}^2)}{\Delta_{mn}} + \frac{K_{mn}}{B_{mn}}\right]; \qquad (9)$$

$$\Delta_{mn} = R^2 \left[\alpha_m{}^4 C_{11}C_{66} + \beta_n{}^4 C_{22}C_{66} + \alpha_m{}^2\beta_n{}^2 (C_{11}C_{22} - C^2{}_{12} - 2C_{12}C_{66})\right]$$

$$K_{mn} = \alpha_m{}^6 C_{13}D_{11}D_{66} + \alpha_m^4 \beta_n^2\left[C_{13}(D_{11}D_{22} - D^2{}_{12} - 2D_{12}D_{66}) + C_{23}D_{11}D_{66}\right] +$$
$$\alpha_m^2\beta_n^4\left[C_{13}D_{22}D_{66} + C_{23}(D_{11}D_{22} - D_{12}{}^2 - 2D_{12}D_{66})\right] + \beta_n{}^6 C_{23}D_{22}D_{66} + C_{13}C_{23} \times$$
$$\left[\alpha_m{}^4 D_{11} + \beta_n{}^4 D_{22} + 2\alpha_m{}^2\beta_n{}^{2_n}(D_{12} + 2D_{66})\right];$$

$$B_{mn} = \alpha_m{}^4 D_{11}D_{66} + \alpha^2{}_m\beta_n{}^2 (D_{11}D_{22} - D^2{}_{12} - 2D_{12}D_{66}) +$$
$$\beta_4{}^4 D_{22}D_{66} + \alpha^2{}_m (C_{13}D_{66} + C_{23}D_{11}) + \beta_n{}^2 (C_{13}D_{22} + C_{23}D_{66}) + C_{13}C_{23}$$

C_{ij} and D_{ij} are hardness of shell [6].

If $C_{13} \to \infty$, $C_{23} \to \infty$ then from equation (8) as per [4] we get below vibration equation based on technical theory:

$$\omega_{mn}^2 = \frac{2}{2\rho h\delta}\left[\alpha_m^4 D_{11} + 2\alpha_m^2\beta_n^2\left(D_{12}+2D_{66}\right)+\beta_n^4 D_{22} + \frac{\alpha_m^4 C_{66}\left(C_{11}C_{22}-C_{12}^2\right)}{\Delta_{mn}}\right]. \quad (10)$$

If we use proved equations of S.A. Ambartsumyan we get as a base of analysis the next values of shear stresses

$$\sigma_{13} = \frac{1}{2}\left(\frac{h^2\delta^2}{a^2} - z^2\right)\varphi(x,y,t);$$

$$\sigma_{23} = \frac{1}{2}\left(\frac{h^2\delta^2}{a^2} - z^2\right)\psi(x,y,t). \quad (11)$$

Based on technical theory we write the displacement and deformation as follows

$$u_1 = u + z\left[-\frac{\partial\omega}{\partial X} + a_{55}\varphi\left(\frac{h^2\delta^2}{a^2 2} - \frac{z^2}{6}\right)\right];\ u_2 = v + z\left[-\frac{\partial w}{\partial y} + a_{44}\psi\left(\frac{h^2\delta^2}{2a^2} - \frac{z^2}{6}\right)\right];$$

$$\varepsilon_{11} = \frac{\partial u}{\partial x} + z\left[-\frac{\partial^2 w}{\partial x^2} + a_{55}\frac{\partial\varphi}{\partial x}\left(\frac{h^2\delta^2}{2a^2} - \frac{z^2}{6}\right)\right];$$

$$\varepsilon_{22} = \frac{\partial v}{\partial y} + \frac{w}{R} + z\left[-\frac{\partial^2 w}{\partial y^2} + a_{44}\frac{\partial\psi}{\partial y}\left(\frac{h^2\delta^2}{2a^2} - \frac{z^2}{6}\right)\right]; \quad (12)$$

$$\varepsilon_{12} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} + z\left[-2\frac{\partial^2 w}{\partial x\partial y} + \left(a_{55}\frac{\partial\varphi}{\partial y} + a_{44}\frac{\partial\psi}{\partial x}\right)\left(\frac{(h\delta)^2}{2a^2} - \frac{z^2}{6}\right)\right].$$

Here $a_{55} = \frac{1}{G_{13}}$, $a_{44} = \frac{1}{G_{23}}$, G_{i3} - shear module.

The intersecting forces and momentums values are defined as follows:

$$N_{13} = \frac{2(hb)^3}{3a^3}\varphi;\ N_{23} = \frac{2(hb)^3}{3a^3}\varphi;$$

$$M_{11} = D_{11}\frac{\partial}{\partial x}\left(-\frac{\partial w}{\partial x} + a_{55}\frac{2(hb)^2}{5a^2}\varphi\right) + D_{12}\frac{\partial}{\partial y}\left(-\frac{\partial w}{\partial y} + a_{44}\frac{2(4b)^2}{5a^2}\varphi\right);$$

$$M_{22} = D_{12}\frac{\partial}{\partial x}\left(-\frac{\partial w}{\partial x} + a_{55}\frac{2(hb)^2}{5a^2}\varphi\right) + D_{22}\frac{\partial}{\partial y}\left(-\frac{\partial w}{\partial y} + a_{44}\frac{2(hb)^2}{5a^2}\varphi\right); \quad (13)$$

$$M_{12} = M_{21} = D_{66}\left(\frac{\partial}{\partial x}\left(-\frac{\partial w}{\partial y} + a_{44}\frac{2(4b)^2}{5a^2}\varphi\right) + \frac{\partial}{\partial y}\left(-\frac{\partial w}{\partial x} + a_{55}\frac{2(4b)^2}{5a^2}\varphi\right)\right).$$

The movement equation of the shell is described with unknowns u, v, w, φ, thus putting the values of intersecting forces and momentums from equation (13) to equation (1), and also considering the values of

$$X_x = \frac{\partial w}{\partial x} + a_{55}\frac{2(4b)^2}{5a^2}\varphi;$$

$$\gamma_y = -\frac{\partial w}{\partial y} + a_{44}\frac{2(4b)^2}{5a^2}\varphi \text{ and}$$

$$C_{13} = K^1\frac{2hb}{a}G_{13}$$

$$G_{23} = K^{11}\frac{2hb}{a}G_{23}$$

$K^1 = K^{11} = 5/6$ we get the equation (9).

This means that the equation taken as per proven theory of S.A. Ambartsumyan is derived as per above equation (9) of netlike shell. If comparing the (ω_{mn}) values of above equation (9) with equation (10), then the relative difference will be the difference of $\delta_{(mn)} = \frac{\omega_{mn}^{(1)}}{\omega_{mn}^{(10)}} - 1$ values.

When R=1, L/R=2 and because m and L values are in combination of $\frac{\pi m R}{L}$ we can analyze the change of m while 1/L is constant, or vice versa. Putting the equation into program we can see that at n=3, n=10 and $G_{12}/G_{13}=1$, δ increases.

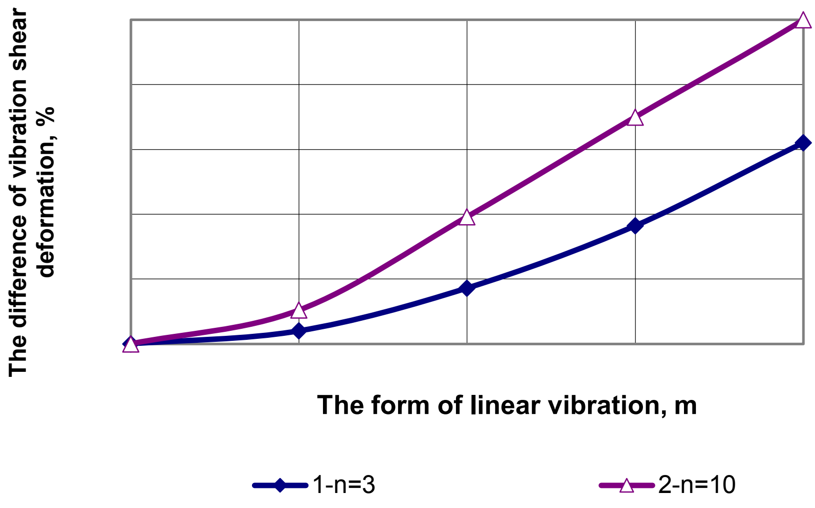

Figure 1 –The graph of changing the difference of vibration using shear deformation, where *n=3, n=10*, from *m*.

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