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Issue Article

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SIMULATION OF GALTON'S BOARD EXPERIMENTS

***Abstract**: The article discusses numerical experiments with a Galton board to simulate various distributions of quantities. It is shown that depending on the parameters of the board model and the falling balls, different distributions can be observed. In conclusion, it is shown that this task (determining the shape of the distribution) is in the general case quite complex.*

***Key words**: Galton board, Galton box, Quincunx, Bean machine, Central limit theorem, Binomial distribution, Standard normal distribution, Python.*

***Language**: English*

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Introduction

The Galton board [1], also known as the Galton box or quincunx or bean machine, is a device invented by Sir Francis Galton in 1874 to demonstrate the central limit theorem [2]. As is well known, if the probability of bouncing right on a peg is p, и n is the number of rows of pegs in a Galton board, then the probability that the ball ends up in the *k*th bin equals $C^k_n p^k (1-p)^{n-k}$. This is the probability mass function of a binomial distribution. According to the central limit theorem, the binomial distribution approximates the normal distribution provided that the number of rows and the number of balls are both large. In this work, we programmed various Galton board models in Python (different shapes of pegs, different sizes of balls and their elasticity coefficients) and showed that it is possible to obtain distributions that look very different from the normal distribution.

Software implementation

For numerical simulation we used Python and the pygame [3] and pymunk [4] libraries. What parameters could be changed in our Galton board model:

1) Cross-sectional shape of pegs: round, square, in the form of equilateral triangles, in the form of right triangles with a slope to the left.

2) Elasticity coefficient ***elasticity*** (you can set its value from 0 to 1).

3) Friction coefficient ***friction***.

4) Board field size ***width*** and ***height***.

5) Board track options: track width ***track_w***, percentage ratio between the width of the track and its wall ***wall_pr***, track height ***track_h***.

6) Pegs parameters: pegs size ***col_rad***, vertical distance between pegs ***col_disty*** and horizontal distance ***col_distx***, number of rows of pegs ***num_lines***.

7) Ball parameters: ball size ***balls_rad***, total number of balls launched ***nums***, ball creation period ***time***.

8) Funnel radius ***out_rad*** (how many times is the radius of the funnel greater than the size of the ball).

9) Specifying the direction and magnitude of gravitational acceleration in a simulation.

10) Number of frames per second ***fps*** and simulation speed ***speed***.

Basic written functions:

1) *create_board* - the function of creating a board consisting of a funnel through which balls will be thrown onto pegs and paths into which the balls should fall;

2) *create_ball* - function of creating a ball with known parameters;

3) *create_circle_col*, *create_square_col*, *create_triangle1_col*, *create_triangle2_col* - functions for creating pegs of different cross-sectional shapes;

4) *calculate_result* - function of counting balls in each track and displaying the result in the form of a histogram.

Simulation results

1. With a relatively large coefficient of elasticity and large round pegs relative to the size of the balls (elasticity = 0.6, col_rad = 15, col_disty = 40, col_distx = 40, num_lines = 11), we obtain a normal distribution shown in Figures 1-2.

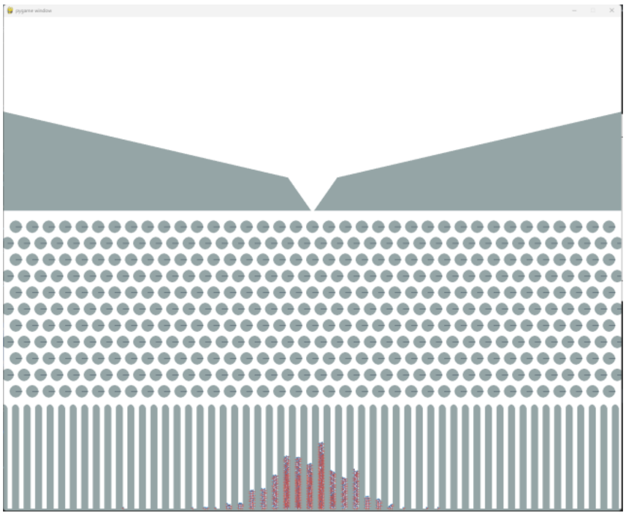

Fig 1. Normal distribution.

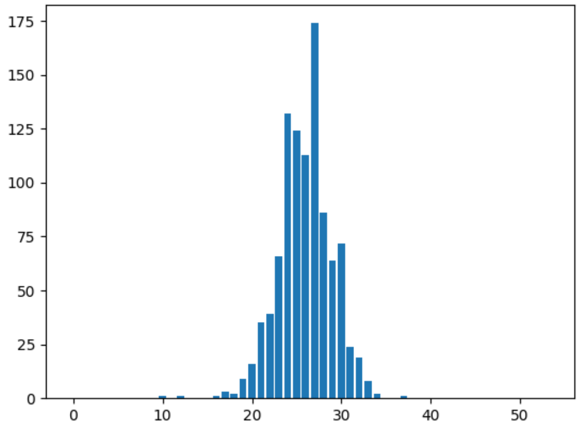

Fig 2. Normal distribution, histogram.

2. With pegs with the shape of a right triangle, the slope of which is directed in one direction (in our case to the left), you can get a distribution similar to the inverse chi-square distribution [5]. Figures 3-4 show the distribution with these parameters: elasticity = 0.6, col_rad = 15, col_disty = 40, col_distx = 40, num_lines = 11; Figures 5-6 show the distribution with these parameters: elasticity = 0.3, col_rad = 15, col_disty = 40, col_distx = 40, num_lines = 11; Figures 7-8 show the distribution with these parameters: elasticity = 0.3, col_rad = 25, col_disty = 40, col_distx = 40, num_lines = 11; Figures 9-10 show the distribution with these parameters: elasticity = 0.6, col_rad = 25, col_disty = 40, col_distx = 40, num_lines = 11.

3. If you choose stakes that are symmetrical relative to the vertical axis, forming a kind of inclined surface, and take a small elasticity coefficient, you can get a distribution similar to the Arcsine distribution [6]. Figures 11-12 show the distribution with round pegs and the following parameters: elasticity = 0.05, col_rad = 15, col_disty = 40, col_distx = 40, num_lines = 11; Figures 13-14 show the distribution with pegs with sections in the form of equilateral triangles and the following parameters: elasticity = 0.05, friction = 0.1, col_rad = 20, col_disty = 40, col_distx = 40, num_lines = 11; Figures 15-16 show the distribution with square pegs and the following parameters: elasticity = 0.1, friction = 0.4, col_rad = 20, col_disty = 40, col_distx = 50, num_lines = 11.

4. If you slightly modernize the Galton board, you can get a distribution similar to the Geometric distribution [7]. To obtain this distribution, you need pegs in the shape of a right triangle with a slope and an additional "wall" located at some distance from the exit of the funnel on the side of the slope (in this case, of course, in the program it is necessary to supplement the function of drawing a board with an operation for drawing such a wall). Figures 17-18 show the resulting distribution with these parameters: elasticity = 0.1, friction = 0.4, col_rad = 15, col_disty = 40, col_distx = 50, num_lines = 11.

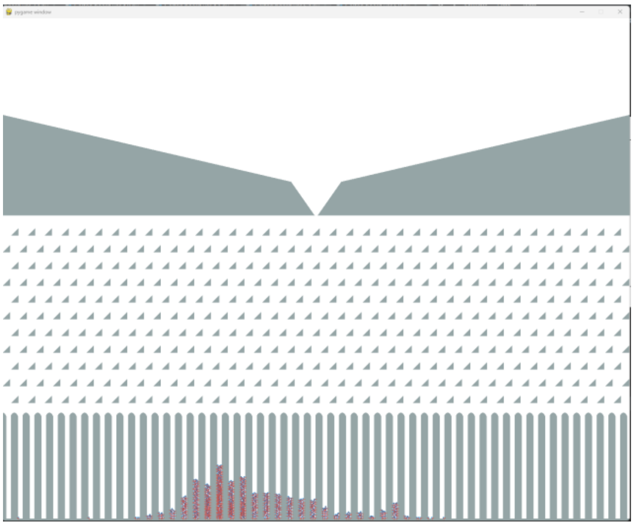

Fig 3. Inverse chi-square distribution, ver. 1.

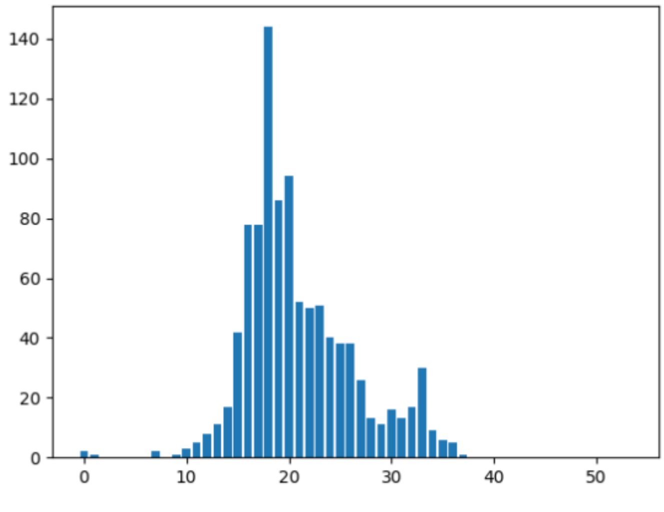

Fig 4. Inverse chi-square distribution, ver. 1, histogram.

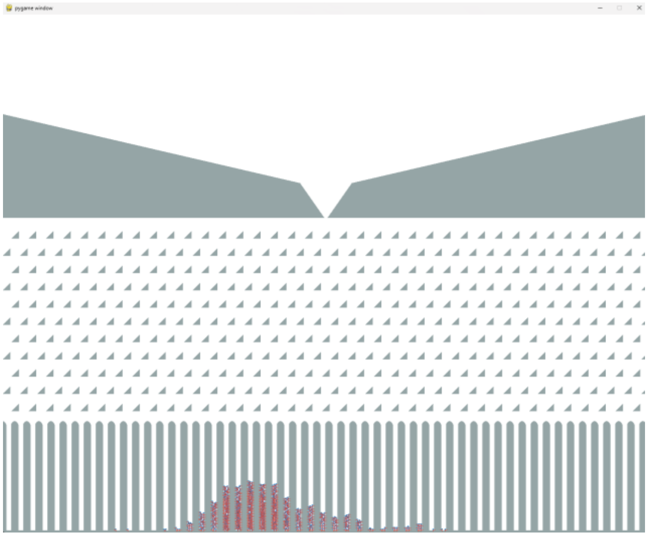

Fig 5. Inverse chi-square distribution, ver. 2.

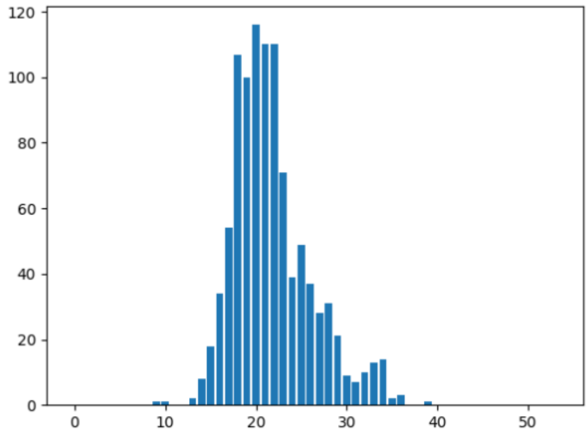

Fig 6. Inverse chi-square distribution, ver. 2, histogram.

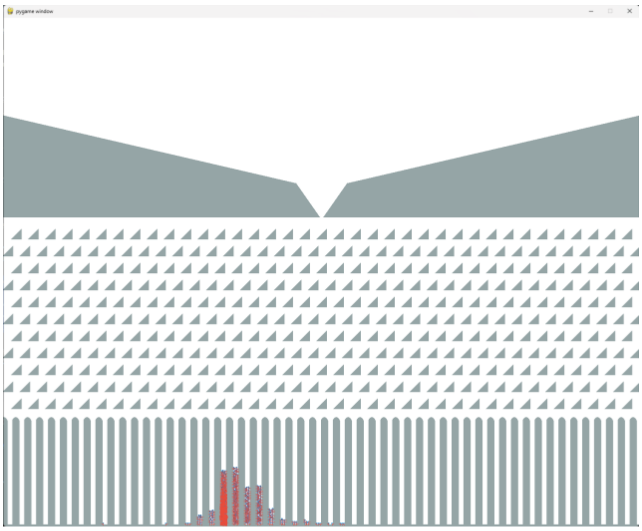

Fig 7. Inverse chi-square distribution, ver. 3.

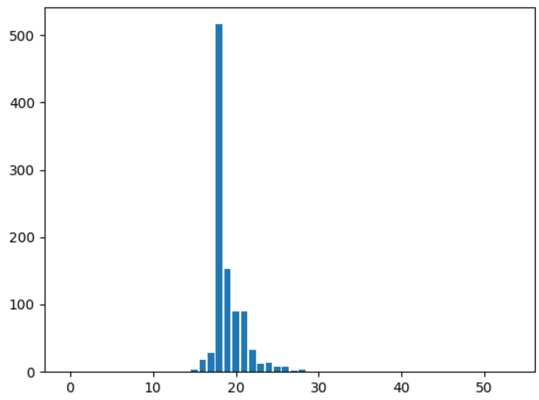

Fig 8. Inverse chi-square distribution, ver. 3, histogram.

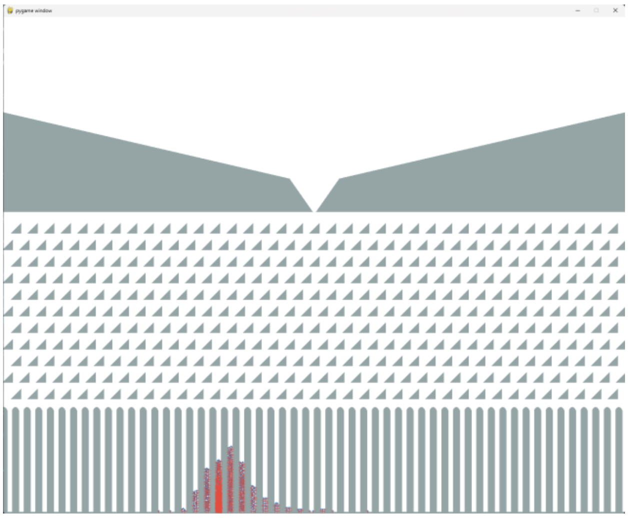

Fig 9. Inverse chi-square distribution, ver. 4.

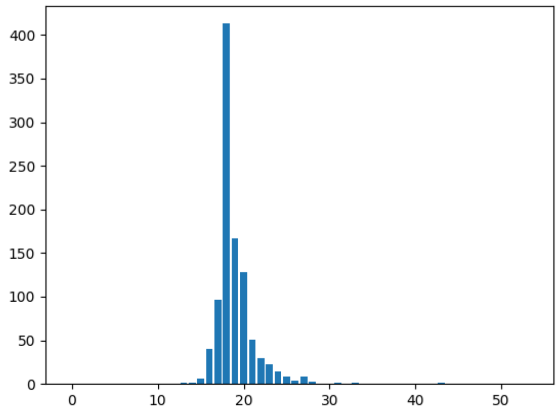

Fig 10. Inverse chi-square distribution, ver. 4, histogram.

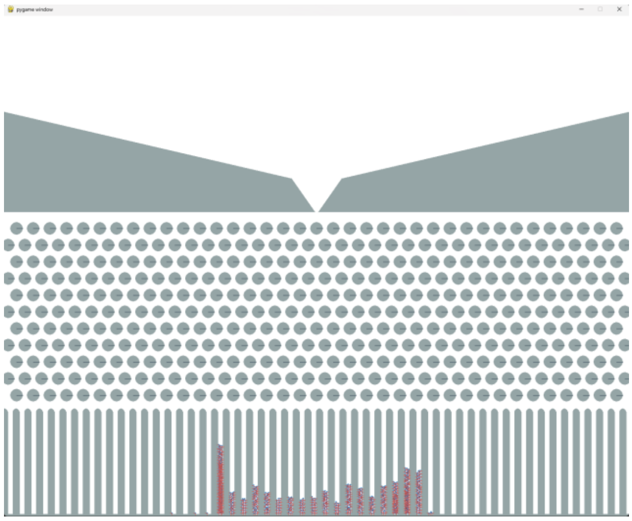

Fig 11. Arcsine distribution, ver. 1.

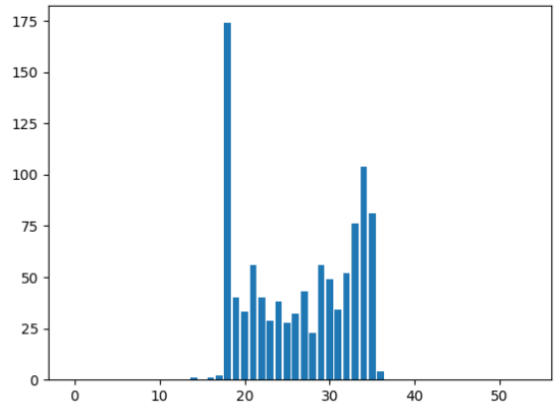

Fig 12. Arcsine distribution, ver. 1, histogram.

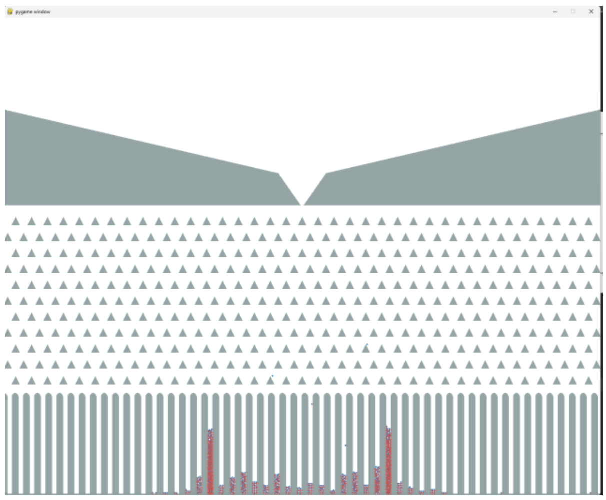

Fig 13. Arcsine distribution, ver. 2.

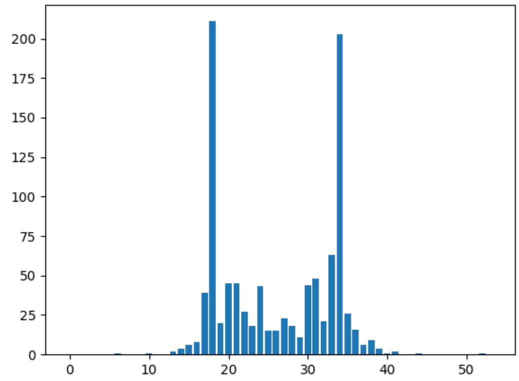

Fig 14. Arcsine distribution, ver. 2, histogram.

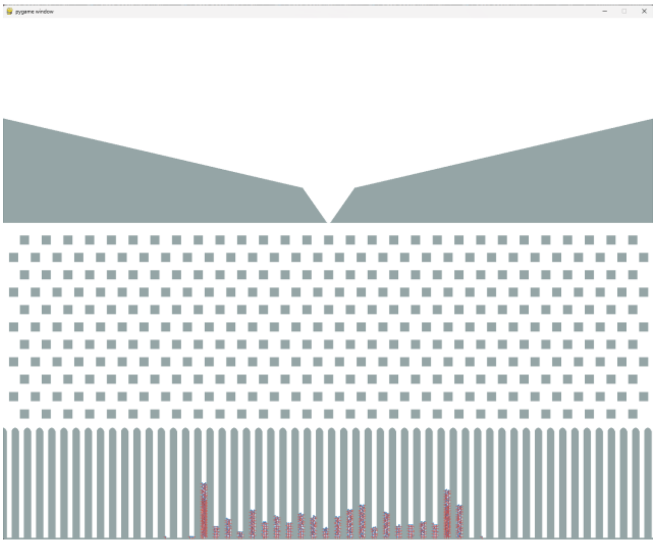

Fig 15. Arcsine distribution, ver. 3.

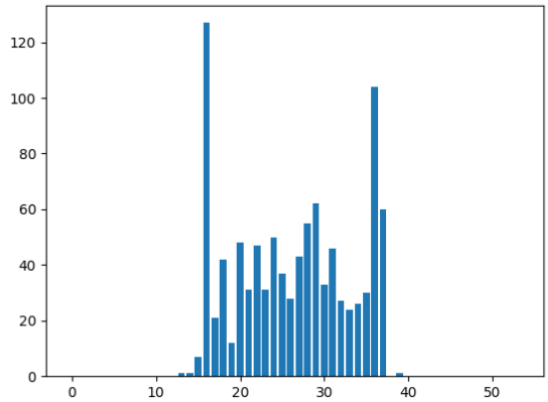

Fig 16. Arcsine distribution, ver. 3, histogram.

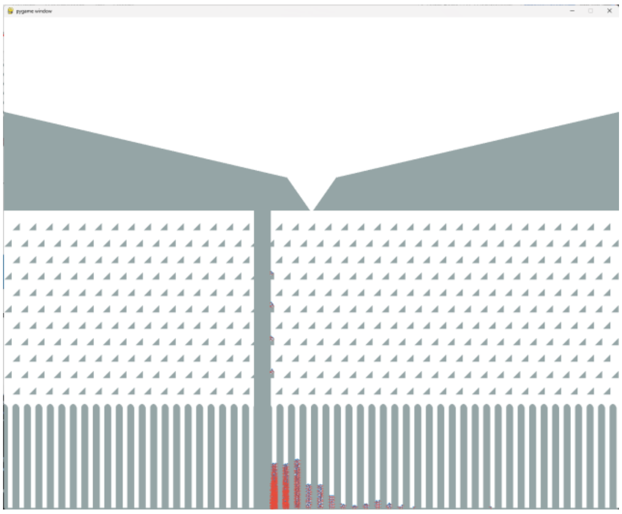

Fig 17. Geometric distribution.

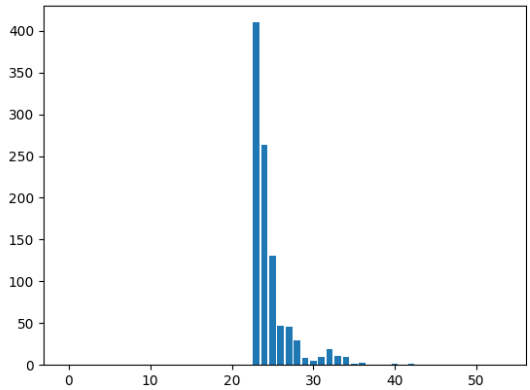

Fig 18. Geometric distribution, histogram.

Conclusion

As can be seen, the simulation results can lead to distributions that are very different from normal. And this is true not only for computer simulations, but also for real (material) Galton boards. It is known that Galton himself spent several months experimenting with different balls and stakes before he selected approximately the geometry and material for a more or less stable demonstration of the normal distribution. Moreover, obtaining various distributions on the Galton board is one of the tasks of the International Physicists' Tournament in 2024 [8] (from there the authors of this work took the idea of conducting such modeling). The fact is that the trajectories of the balls are nonlinear, and the type of distribution as a result depends on the relationship of many parameters in the problem. If we assume that when a ball collides with a peg, the law of conservation of energy and momentum is satisfied and the angle of reflection is equal to the angle of incidence (this is an ideal reflection), then this problem is similar to mathematical billiards, but not with straight trajectories, but with parabolic ones (due to gravity). Several articles are devoted to this kind of billiards - see, for example, [9]. Moreover, a similar consideration is also applicable in real physics - see, for example, [10]. Under the above assumptions about the ideality of collisions and pegs of an ideal circular cross-section, the problem of calculating trajectories can be solved analytically - since when the intersection of a parabolic trajectory with a circle is sought, an algebraic equation of the fourth degree is obtained, and an algebraic equation of the fourth degree can be solved analytically. But in a real board, the collision will never be ideal, and the shapes of the pegs are also not ideal regular figures, so this problem can no longer be solved analytically. Actually, one of the goals of this work was to show that the example with Galton’s board, often found in textbooks of mathematics and physics, as an illustration of obtaining a normal distribution, is only a simplified case.

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