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Issue Article

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DIFFRACTION OF A HARMONIC WAVE BY A REINFORCED HOLE WITH LIQUID

***Abstract**: The paper considers an infinitely long circular cylinder, consisting in the general case of a finite number of coaxial viscoelastic layers, surrounded by a deformable medium. The dynamic stress-strain state of a piecewise homogeneous cylindrical layer from a harmonic wave is investigated. Numerical results of stresses depending on the wavelength are obtained.*

***Key words**: Viscoelasticity, liquid, frequency, longitudinal and transverse wave, shell, plane strain, numerical results.*

***Language**: English*

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Introduction

In the world, scientific research on the study of the problem of wave propagation in structures consisting of long bodies of variable thickness with elastic and viscoelastic properties is carried out in more than one hundred research institutes [1, 2, and 3]. Many parts of modern moving vehicles operate with the risk of resonance caused by vibration loads. In order to ensure their durability and reduce the stress-strain state, funds are spent annually in the range of 3-5 billion US dollars. In order to ensure permissible limits of stress and deformation, as well as dynamic processes in viscoelastic plate and shell bodies of variable thickness, which are an integral part of such mechanisms, it is necessary to study the process of wave propagation in them [4,5]. In this direction in the USA, Germany, Russia, Israel, South Korea, China and other industrially developed countries in order to ensure the growth of durability and efficiency of materials used in technology, to ensure their comprehensive convenience, efficiency, special attention is paid to the development of design solutions that increase their service life, the introduction of economical and optimal technical and technological designs consisting of viscoelastic plate and shell bodies with variable thickness [6,7]. But, despite this, there are a number of unresolved issues in the study of wave propagation in waveguides with a variable cross section, taking into account the rheological properties of the material, as well as the presence of various cracks. In addition, the scientific basis for wave propagation in dissipative inhomogeneous bodies consisting of flat parallel layers has not been developed [8,9]. The problem of developing a methodology and programs for studying wave propagation in dissipative inhomogeneous long bodies, as well as solving new problems has not been solved [10,11]. A number of such problems also include: the choice of the relaxation kernel and its parameters, which represent the viscoelastic properties of waveguides, as well as the comparison and application of various existing theories (approximate and exact), based on the analysis of dispersion relations; the introduction and justification of a value representing the attenuation of a wave, the creation of a unified methodology and program that solves the tasks set, the study of the influence of geometric parameters on various modes of phase velocities.

Statement of the problem and basic relationships. There are many underground tunnels of various shapes located in seismically active areas that need to be protected from seismic impacts. The paper considers the impact of harmonic waves on a cylindrical shell located in a viscoelastic half-plane. The main objective of the study is to determine the stress-strain state of the cylindrical shell under the influence of harmonic waves. The main equation of viscoelasticity in displacements with the corresponding boundary conditions is obtained. The problem is solved in mixed potentials that satisfy the wave equation with complex parameters. The solution is expressed through special Bessel and Hankel functions. As a result of multiple reflection, a system of algebraic equations with complex coefficients is obtained. In the future, this system is solved by the Gauss method with the allocation of the main element. The analytical solution is obtained in infinite series, the convergence of which is studied numerically. Numerical results are obtained on the MATLAB software package. The reliability of the obtained research results is confirmed by good agreement with theoretical and experimental results and those obtained by other authors [12]

$$(\tilde{\lambda}_j + 2\tilde{\mu}_j) grad\,(div(\vec{u}_j)) - \tilde{\mu} rot(rot(\vec{u}_j)) + \vec{b}_j = \rho_j \frac{\partial^2 \vec{u}_j}{\partial t^2},$$

where λ_j and μ_j ($j = 1,2 ......$N, - relate to the environment, - to the layer) are the operator elastic moduli[13]

$\vec{b}_j$ – vector of the density of volumetric forces $(b_j = 0)$; $f(t)$ – some function; ρ_j – material densities, $R_\mu^{(i)}(t-\tau)$ and $R_\lambda^{(i)}(t-\tau)$ -relaxation kernel, λ_{oj}, μ_{oj} -instantaneous elastic moduli of the viscoelastic material, $\vec{u}_j(u_{rj}, u_{\theta j})$ -displacement vector, which depends on r, θ, t. At pressures up to 100 MPa, the motion of the liquid is satisfactorily described by wave potentials for the velocity of liquid particles [12]

$$\Delta\varphi_0 = \frac{1}{C_o^2}\frac{\partial^2 \varphi_0}{\partial t^2}, \qquad (2)$$

Where $\Delta = \frac{d^2}{dr^2} + \frac{d}{rdr} + \frac{d^2}{r^2 d\theta^2}$; Co - is the acoustic speed of sound in the liquid. The potential φ_0 and the liquid velocity vector are related by the dependence $\vec{V} = grad\varphi_0$. The liquid pressure is determined using $r = R_0$ the linearized Cauchy-Lagrange integral $P = -\rho_o C_0 \frac{\partial \varphi_0}{\partial t}$ - the liquid pressure on the wall of the cylindrical layer and ρ_o is the liquid density. Under the condition of continuous flow of the liquid, the normal component of the liquid velocity and the layer on the surface of their contact $r = R_0$ should be equal

$$\left.\frac{\partial \varphi_0}{\partial r}\right|_{r=R_0} = \left.\frac{\partial u_{r1}}{\partial t}\right|_{r=R_0}, \qquad (3)$$

where u_{r1} – displacements of the layer along the normal. At the contact of two bodies $r = R_j$, the equality of displacements and stresses is fulfilled (rigid contact condition)

$$u_{rj} = u_{r(j+1)};\quad \sigma_{rrj} = \sigma_{rr(j+1)}$$

$$u_{\theta j} = u_{\theta(j+1)};\quad \sigma_{r\theta j} = \sigma_{r\theta(j+1)}. \qquad (4)$$

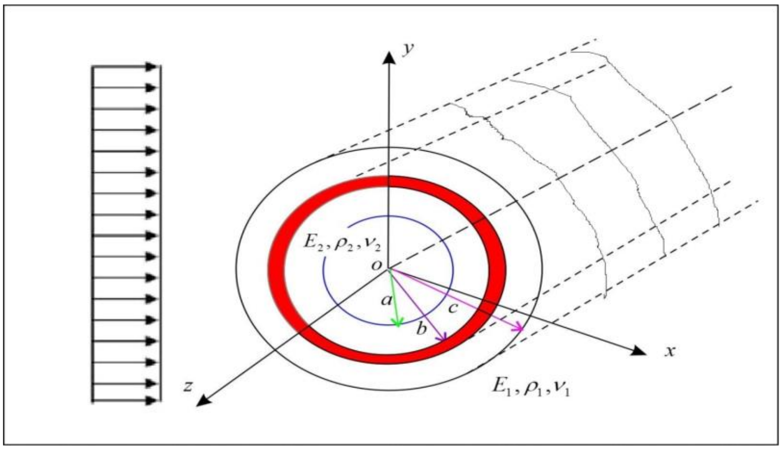

Fig. 1. Calculation scheme of layered cylindrical bodies located in a deformable medium

Note that in the case of sliding contact of the soil along the surface of the pipe, the last equation in (4) takes the form [13]: $\sigma_{r\theta j}=0$, where, $\sigma_{nn}^{(j)}$ and $\sigma_{ns_1}^{(j)}$ are the radial and tangential stresses in the j-th viscoelastic body; and are the radial and tangential displacements of the j - th body.

A plane longitudinal wave with potential falls $\phi^{(p)}$ on a cylindrical shell with radius R, the frequency and amplitude of the incident waves are in ϕ_0 the form

$$\phi^{(p)}=\phi_0 e^{i(\vec{\alpha}\cdot\vec{r}-\omega t)},\psi^{(p)}=0 \qquad (5)$$

where $\vec{r}=r(\cos\theta_0\vec{i}+\sin\theta_0\vec{j})$, $\vec{\alpha}=\alpha(\cos\gamma_0\vec{i}+\sin\gamma_0\vec{j})$, $\alpha=\omega/c_{p1}$, θ_0-is the angle of the cylindrical shell in γ_0 the main coordinate system, is $\vec{i},\vec{j}$ the angle of inclination of the incident loads, are unit vectors along the x and y axes, respectively (Fig. 1). Further, $\phi^{(p)}$ the incident wave in cylindrical coordinates has the form [14]

$$\phi^{(p)}=\phi_0 e^{i\alpha_1 r\cos\gamma_0+i\omega t}\sum_{n=-\infty}^{\infty}J_n(\alpha_{L1}r)e^{in(\theta+\gamma_0)}. \qquad (6)$$

The scattering of a wave of order m can be expressed through the Hankel functions of the first kind of the nth order of the complex argument

$$\phi_1^{(sm)}=\sum_{n=-\infty}^{\infty}\left[A_n^{sm}H_n^{(1)}(\alpha_{L1}r)+\bar{A}_n^{sm}H_n^{(2)}(\alpha_{L1}r)\right]e^{in(\theta_0-\gamma)-i\omega t}, \qquad (7)$$
$$\psi_1^{(sm)}=\sum_{n=-\infty}^{\infty}\left[B_n^{sm}H_n^{(1)}(\beta_{M1}r)+\bar{B}_n^{sm}H_n^{(2)}(\beta_{M1}r)\right]e^{in(\theta_0-\gamma)-i\omega t}.$$

Here A_n^{sm} and B_n^{sm} are the complex scattering coefficients of order m, are the Hankel functions of the first kind of the n-th order of the complex argument. The coefficients $A_n^{sm},\bar{A}_n^{sm}$, B_n^{sm}, $\bar{B}_n^{sm}$, are determined from the boundary conditions, which in the case under consideration have the form:

$$\sigma_{rrm}^{(k)}=\sigma_{rrm}^{(k+1)},\quad \sigma_{r\theta m}^{(k)}=\sigma_{r\theta m}^{(k+1)},$$
$$u_{rm}^{(k)}=u_{rm}^{(k+1)},\quad u_{\theta m}^{(k)}=u_{\theta m}^{(k+1)}. \qquad (8)$$

Similarly, for each incident and scattering harmonic wave, the Somerfield radiation conditions are satisfied.

The solution of equation (1), for cylindrical shells, is expressed through the Hankel functions of the first and second kind of the nth order:

$$\phi_2^{(s)}=\sum_{n=-\infty}^{\infty}\left[C_n^{sm}H_n^{(1)}(\alpha_{L2}r)+D_n^{sm}H_n^{(2)}(\alpha_{L2}r)\right]e^{in(\theta_0-\gamma)-i\omega t}, \qquad (9)$$
$$\psi_2^{(s)}=\sum_{n=-\infty}^{\infty}\left[L_n^{sm}H_n^{(1)}(\beta_{M2}r)+M_n^{sm}H_n^{(2)}(\beta_{M2}r)\right]e^{in(\theta_0-\gamma)-i\omega t},$$

where C_n^{sm}, D_n^{sm}, L_n^{sm}, M_n^{sm} are the expansion coefficients, which are determined by the corresponding boundary conditions $H_n^{(1)}(\alpha_{L2}r),H_n^{(2)}(\alpha_{L2}r)$, $H_n^{(1)}(\beta_{M2}r)$ and $H_n^{(2)}(\beta_{M2}r)$ are, respectively, the Hankel function of the 1st and 2nd kind of the nth order $H_n^{(2)}(\alpha r)=I_n(\alpha r)-iN_n(\alpha r)$.

$$u_{rk}=\frac{\partial\phi_k}{\partial r}+\frac{1}{r}\frac{\partial\psi_k}{\partial\theta};u_{\theta k}=\frac{1}{r}\frac{\partial\phi_k}{\partial\theta}-\frac{\partial\psi_k}{\partial r},$$
$$\varepsilon_{rrk}=\frac{\partial u_{rk}}{\partial r};\varepsilon_{\theta\theta k}=\frac{1}{r}\frac{\partial u_{\theta k}}{\partial\theta}+\frac{u_{rk}}{r};\varepsilon_{r\theta k}=\frac{1}{2}\left(\frac{1}{r}\frac{\partial u_{rk}}{\partial\theta}+\frac{\partial u_{\theta k}}{\partial r}+\frac{u_{\theta k}}{r}\right), \qquad (10)$$
$$\sigma_{rrk}=\tilde{\lambda}_k\nabla^2\phi_k+2\tilde{\mu}_k\left[\frac{\partial^2\phi_k}{\partial r^2}+\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial\phi_k}{\partial\theta}\right)\right],$$
$$\sigma_{\theta\theta k}=\tilde{\lambda}_k\nabla^2\phi_k+2\tilde{\mu}_k\left[\left[\frac{1}{r}\left(\frac{\partial\phi_k}{\partial r}+\frac{1}{r}\frac{\partial^2\phi_k}{\partial\theta^2}\right)+\frac{1}{r}\left(\frac{1}{r}\frac{\partial\psi_k}{\partial\theta}-\frac{\partial^2\psi_k}{\partial r\partial\theta}\right)\right]\right],$$
$$\sigma_{zz}=\tilde{\lambda}_k\nabla^2\phi_k;\sigma_{r\theta k}=2\tilde{\mu}_k\left(\frac{1}{r}\frac{\partial^2\phi_k}{\partial\theta\partial r}-\frac{1}{r^2}\frac{\partial\phi_k}{\partial\theta}\right).$$

where are $\varepsilon_{rr}, \varepsilon_{r\theta}, \varepsilon_{\theta\theta}$ the elements of the strain tensor; are the elements of $\sigma_{rr}, \sigma_{r\theta}, \sigma_{\theta\theta}, \sigma_{zz}$ the stress tensor.

Results and discussions. Let us consider the problem of wave scattering on an elastic half-plane from circular shells. We assume that the tunnel structure is a steel shell with a radius $R_1 = 1.75, R_2 = 2.0,\ h = 0.25,\ \theta_0 = 0^0$, H/R=2.0. The $R(t) = Ae^{-\beta t}/t^{1-\alpha}$ three-parameter Koltunov-Rzhanitsyn relaxation kernel was used in the calculations: $A = 0,048; \beta = 0,05; \alpha = 0,1$, with parameters:

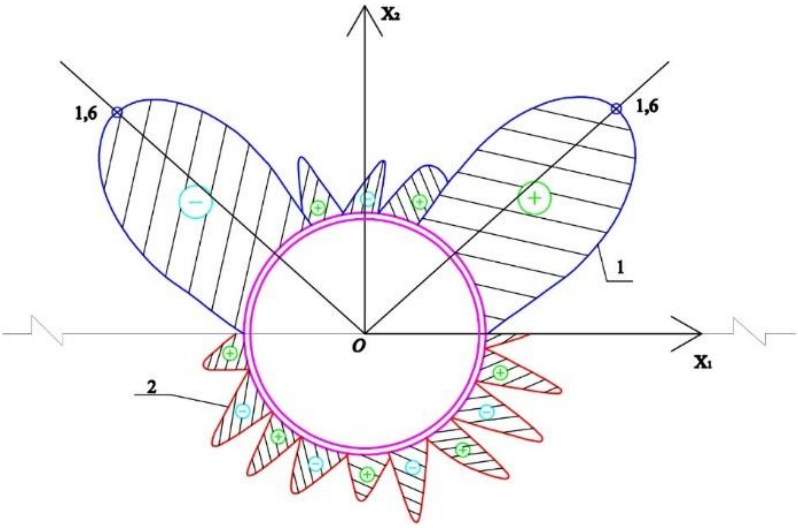

Figure 2. Diagrams of contour stresses of the shell at different frequencies:
$1.\ \omega = 20Hz;\quad 2.\ \omega = 40Hz;\quad 3.\ \omega = 80Hz$.

The calculation results are shown in Figure 2 and Figure 3 for the effect of transverse waves. The $\omega \in [19,30]Hz$ maximum contour stresses in the shell were obtained at frequency values . Figure 2 shows that the effect of the source proximity consists of moving the maximum value to the point where the straight line drawn from the source touches the cavity boundary.

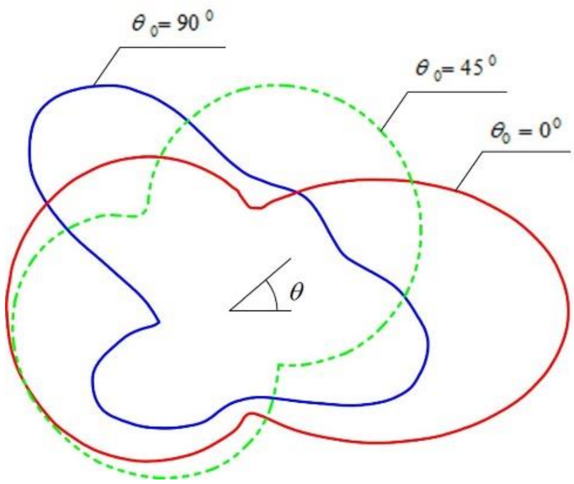

Fig. 3. Diagrams of contour stresses of the shell at different frequencies:
$1.\ \omega = 20Hz;\ 2.\ \omega = 40Hz;\ 3.\ \omega = 80Hz$.

For stress concentration coefficients, we will use the absolute value of the complex quantity

Conclusions

1. The problem of harmonic wave diffraction in a cylindrical body is solved in displacement potentials. Displacement potentials are determined from solutions of the Helmholtz equation. Arbitrary constants are determined from the boundary conditions that are set between the bodies. As a result, the problem is reduced to a system of inhomogeneous algebraic equations with complex coefficients that are

solved by the Gauss method with the selection of the main element.

2. Contour stresses σθθ on the free surface of cylindrical bodies reach their maximum value of 900-when exposed to longitudinal waves, and 450-when exposed to transverse waves. Contour stresses σθθ when exposed to transverse harmonic waves are 15-20% greater than when exposed to longitudinal waves.

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