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Issue Article

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GENERALIZED FORMULAS FOR FRICTION FORCE AND CONSTRAINT REACTION FORCE OF A HORIZONTAL AXISYMMETRIC WEDGE

Abstract*: The article studies the static state of an axisymmetric wedge. The static problem is solved using classical mechanics methods. The constraints are replaced by reaction forces. The equations of the wedge equilibrium state are derived. Generalized formulas for the friction force and the constraint reaction force of the wedge are obtained. A comparative analysis of the calculations is conducted.*

Key words*: wedge, friction force, constraint reaction forces, equilibrium equation.*
Language*: English*
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Introduction

Wedge-shaped working bodies are used in many industries. Determining the unknown friction forces and constraint reaction forces of the wedge is a relevant problem.

A generalized model of a wedge-shaped spring-friction set of a freight car bogie was developed in [1]. Critically discussing the lack of classical mechanics methods in numerous publications, the equation of equilibrium of friction wedges was derived according to the principle of release from restraints and an analytical expression of the reaction of a friction wedge was obtained [2]. A mathematical model of a wedge clutch was developed considering sliding friction [3].

A theory of a dihedral wedge widely used in agricultural research was created in [4]. Calculations were conducted based on this theory and the results obtained were compared, certifying the reliability of the formulas derived by this theory [5].

In this article, generalized formulas for the friction force and the restraint reaction force of an axisymmetric wedge are obtained.

Material

Consider the statics of penetration of a horizontal axisymmetric wedge into a homogeneous medium under constant force $\bar{P}$. We determine the friction and reaction forces of the wedge using methods of theoretical mechanics [6, 7].

On the wedge surfaces in contact with the medium, friction forces $\bar{F}_{1fr}$ and $\bar{F}_{2fr}$, directed along the working planes of the wedge, are formed. The reaction forces of the medium $\bar{N}_1$ and $\bar{N}_2$ are directed perpendicular to these planes (Fig. 1).

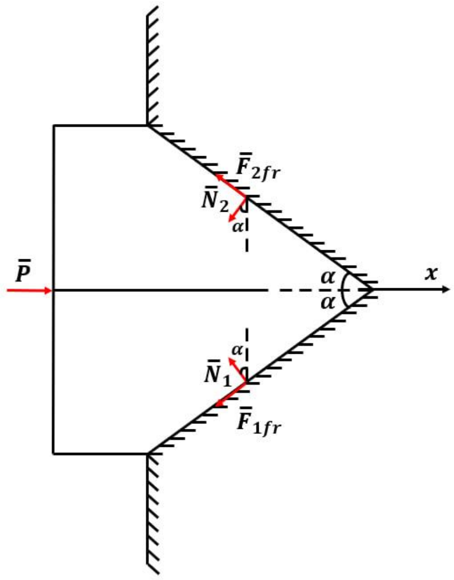


Fig. 1. Wedge shape and forces applied to it

Let us derive an equilibrium equation for the wedge, equating to zero the sum of all projections of the forces applied to the wedge on the *x*-axis:

$$P - N_1 \sin\alpha - F_{1fr}\cos\alpha - N_2 \sin\alpha - F_{2fr}\cos\alpha = 0. \quad (1)$$

From (1), with equalities $N_1 = N_2 = N$, $F_{1fr} = F_{2fr} = F_{fr}$ for an axisymmetric wedge and considering the Coulomb formula $F_{fr} = fN$ [6], we obtain $fP = 2F_{fr}(\sin\alpha - 2f\cos\alpha)$.

From here, we determine the friction force and, with $F_{fr} = fN$, the constraint reaction force of the wedge:

$$F_{fr} = \frac{fP}{2(\sin\alpha + f\cos\alpha)}, \quad (2)$$

$$N = \frac{P}{2(\sin\alpha + f\cos\alpha)}. \quad (3)$$

Here, f - is the friction coefficient.

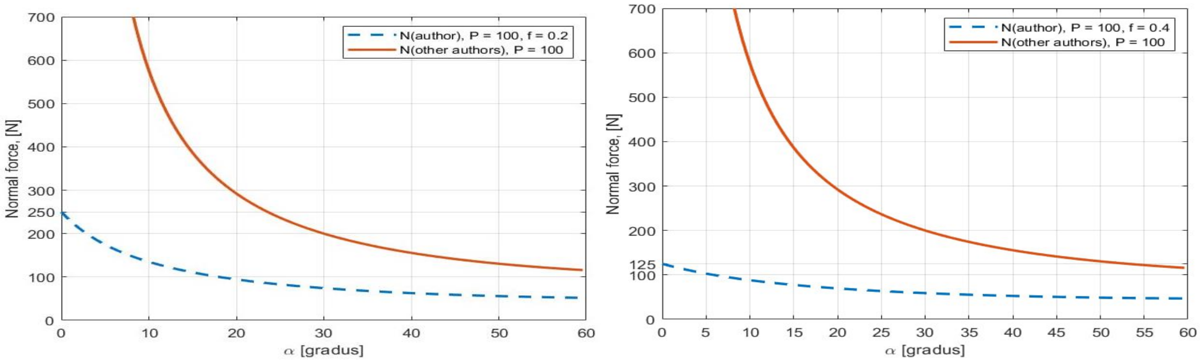
Thus, expressions for the friction force and the restraint reaction force of the horizontal axisymmetric wedge are obtained. Using these formulas, we can determine the values of the friction and reaction forces at an arbitrary angle of inclination.

Results.

Using the solutions obtained, calculations were conducted for values $f = 0,2$ (coefficient of friction between steel and crushed stone) and $f = 0,4$ (coefficient of friction between steel and soil) for different values of applied force $\bar{P}$.

Figure 2 shows the results of calculating the constraint reaction force depending on the angle of inclination of the wedge at an applied force $P = 100$ *N*. The graphs in Fig. 2*a* were plotted for $f = 0,2$;

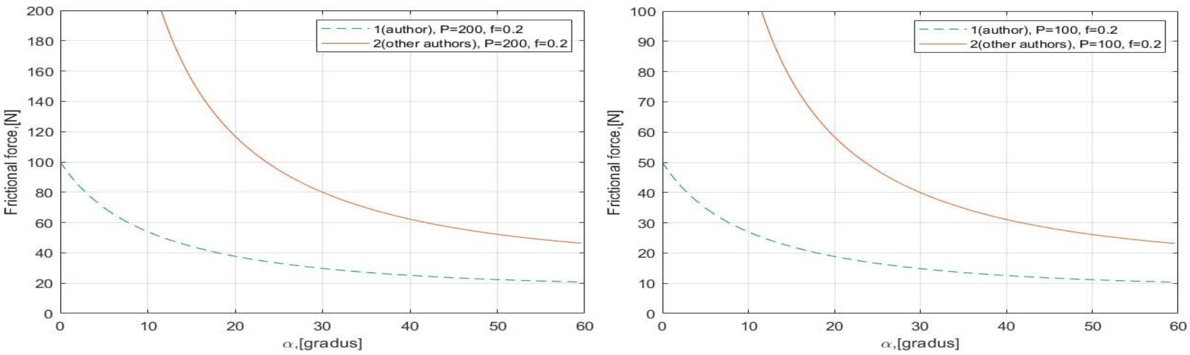
and the graphs in Fig. 2*b* correspond to $f = 0,4$. The graph with a solid line was obtained for the value of the reaction force by formula $N = P / \sin\alpha$ [6, 7, 8, 9] and from a wedge inclination angle of $\alpha = 10^0$, its value tends to infinity. Another line was obtained by the formula derived by the authors of this article (3), according to which the reaction force has a numerical value at any value of the angle. A two times increase in the friction coefficient led to an increase in the value of the reaction force in the corresponding ratio.



a) *b*)

Fig. 2. Change in the reaction force depending on the angle of inclination of the wedge.

Figure 3 shows the results of friction force calculations for the value of the friction coefficient $f = 0,2$ for $P = 200$ N (*a*) and for $f = 0,2$; $P = 100$ N (*b*). The solid line is obtained by formula $F_{fr} = Pf / \sin\alpha$ [6, 7, 8, 9] and it is clear that for $\alpha = 0$, the value of the friction force tends to infinity. The other line is obtained by formula (2) and here the friction force has a specific numerical value.



a) *b*)

Fig. 3. Change in the friction force depending on the angle of inclination of the wedge.

Conclusion.

Analytical expressions for the friction and reaction forces of an axisymmetric wedge are obtained. These forces have specific numerical values for all values of the angle of inclination of the wedge. Solutions obtained present generalized expressions of the friction force and the constraint reaction force of an axisymmetric wedge.

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