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Issue Article

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DIFFRACTION OF A HARMONIC WAVE PROPAGATING THROUGH AN ELLIPTICAL CYLINDER IN A VISCOELASTIC MEDIUM

***Abstract**: The paper solves the problem of the boundary of an elliptical gap in an unbounded viscoelastic medium and the stress-strain state that arises around it when a shear wave is applied. The problem is formulated in an elliptical coordinate system and reduced to the Mathieu equation, and the solution is obtained using the Mathieu function. Numerical results are obtained in the Matlab program.*

***Key words**: shear wave, stress-strain state, elliptical coordinate system, Mathieu function.*
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Introduction

The problem of studying the interaction of underground structures with the soil is relevant [1,2,3]. When studying this problem, underground structures can have different cross-sections. In recent years, the construction of underground communications in seismically active areas has been developing at a rapid pace. Therefore, the calculation of the strength of such structures (pipelines, tunnels, underground passages, etc.) under the influence of seismic and blast waves, and the study of the stress-strain state is a pressing task [4,5]. The following methods for calculating seismic and explosion resistance of underground structures are considered [6,7]. There are two main methods for studying the stress state of underground structures under the influence of seismic waves: experimental studies and theoretical research methods. The problem of the stress-strain state arising at the boundary of an elliptical gap in an unbounded viscoelastic medium under the influence of a shear wave load is solved in an elliptical coordinate system (Fig. 1). Let the incident wave form a wave with an abscissa axis, as shown in the figure. The main goal is to study dynamic stresses and deformations depending on geometric and physical-mechanical parameters. Such a problem is presented in [8] for an elastic medium. The difference from their work is that here the elastic property of the medium is taken into account. Numerical calculations were carried out for different frequencies, angles of incidence of the shear wave and ratios of the ellipse axes.

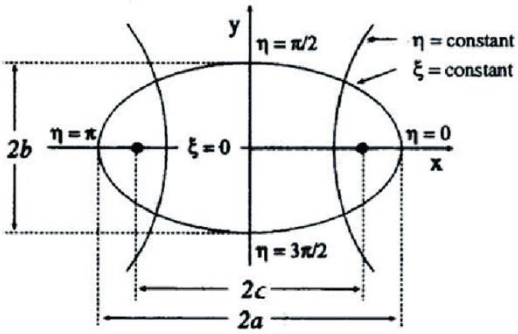

Figure 1. Shear harmonic wave load on an elliptical cylinder in a homogeneous elastic medium.

The problem under consideration is reduced to the problem of the deformed state of the theory of viscous elasticity. In an elliptical coordinate system, the stress takes the form

$$\sigma_{\xi x_3} = \frac{\bar{\mu}}{H}\frac{\partial u_\theta}{\partial \xi}, \sigma_{\eta x_3} = \frac{\bar{\mu}}{H}\frac{\partial u_\theta}{\partial \eta}, \qquad (1)$$

where is the shear modulus $\bar{\mu}$, a complex value;

$H = c\sqrt{ch^2\xi - \cos^2\eta};$ 2c-focal length, u_θ - displacement in displacement satisfying the Helmholtz equation below

$$\frac{1}{H^2}\left(\frac{\partial^2 u_\theta}{\partial \xi^2} + \frac{\partial^2 u_\theta}{\partial \eta^2}\right) + k_s^2 u_\theta = 0. \qquad (2)$$

where $k_s^2 = \omega^2 \rho / \mu_0 \Gamma_\mu$, ω - frequency of the incident wave, real value, ρ - material density, Γ_μ - a parameter expressing the viscosity of a material (medium). If $\Gamma_\mu = 1$, then the medium is elastic. For elastic materials, the connection is a complex quantity. However, the moduli of the real and imaginary parts are less than one and greater than zero. The shear stress on the inner surface of an elliptical cylindrical cavity is zero.

$$\sigma_{\xi x_3}\big|_\Gamma = 0 \qquad (3)$$

An elliptical void surface in a viscoelastic medium is bounded by a line $\Gamma(\xi = \xi_0, 0 \le \eta \le 2\pi) -$ We can take the falling harmonic external wave as shown below

$$u_{\theta 0} = A_p \exp\left[ik(x\cos\theta + y\sin\theta)\right], \qquad (4)$$

where A_p - amplitude of the incident wave, $k = k_s c / 2$. Expression (4) is expressed in a Cartesian coordinate system. If this equality is expressed in an elliptical coordinate system, it will take the following form:

$$u_{\theta 0} = A_p \sum_{n=0}^{\infty} i^n \left[\begin{array}{l} Ce_n(\xi,q)ce_n(\eta,q)ce_n(\theta,q) + \\ +Se_{n+1}(\xi,q)se_{n+1}(\eta,q)se_{n+1}(\theta,q) \end{array}\right] \qquad (5)$$

Problem statement and solution method

The solution of the above partial differential equation (3) with complex coefficients is reduced to the Mathieu equation, and the solution is expressed by the Mathieu function

$$u_\theta = \sum_{n=0}^{\infty}\left[\begin{array}{l} C_n Me_n^{(1)}(\xi,q)ce_n(\eta,q)ce_n(\theta,q) + \\ +S_{n+1}Ne_{n+1}^{(1)}(\xi,q)se_{n+1}(\eta,q)se_{n+1}(\theta,q) \end{array}\right]$$

$$q = k^2 \qquad (6)$$

where are trigonometric functions $Ce_n(\xi,q), Se_n(\xi,q) -$ radial Mathieu functions. $Me_n^{(1)}(\xi,q)$, $Ne_{n+1}^{(1)}(\xi,q)$ - Mathieu-Hankel functions in the form of a series taking into account the backward wave, C_n, S_{n+1} - arbitrary constant values. u_θ the solution satisfies the Sommerfeld fading condition. On the other hand, we use the boundary condition (3) to find arbitrary constants in the solution (6)

$$\sum_{n=0}^{\infty}\left[\begin{array}{l} C_n Me_n'(\xi_0,q)ce_n(\eta,q)ce_n(\theta,q) + \\ +S_{n+1}Ne_{n+1}'(\xi_0,q)se_{n+1}(\eta,q)se_{n+1}(\theta,q) \end{array}\right] =$$

$$= -A_p \sum_{n=0}^{\infty} i^n \left[\begin{array}{l} Ce_n'(\xi_0,q)ce_n(\eta,q)ce_n(\theta,q) + \\ iSe_{n+1}'(\xi_0,q)se_{n+1}(\eta,q)se_{n+1}(\theta,q) \end{array}\right] \qquad (7)$$

If we multiply both sides of equation (7) by $ce_0(\eta,q), ce_1(\theta,q), ...$ and integrate along the line of the ellipse's circumference, using the orthogonality condition of the Mathieu function, then we find an arbitrary constant C_n

$$C_n = i^n A_p Ce_n'(\xi_0,q) / Me_n(\xi_0,q). \qquad (8)$$

If we use $se_{n+1}(\eta,q)$, that

$$S_{n+1} = -i^{n+1} A_p Se_{n+1}'(\xi_0,q) / Ne_{n+1}(\xi_0,q). \qquad (9)$$

If the found arbitrary constants are substituted into the solution, it will be reduced to the form shown below.

$$u_\theta = A_p \sum_{n=0}^{\infty} i^n \left\{ \begin{array}{l} \left[Ce_n(\xi,q) - Me_n(\xi,q)Ce_n(\xi_0,q)/\right. \\ \left. Me_n(\xi_0,q)\right] \times ce_n(\eta,q)ce_n(\theta,q) \times \\ +i\left[Se_{n+1}(\xi\ ,q) - \right. \\ \left. -Ne_{n+1}(\xi,q)Se_{n+1}(\xi_0,q)\right]/ \\ Ne_{n+1}(\xi_0,q)se_{n+1}(\eta,q)se_{n+1}(\theta,q) \end{array} \right\} \qquad (10)$$

Knowing the displacement, we can find the forces (1) using (10)

$$\sigma_{\eta x_3} = (A_p / J)\sum_{n=0}^{\infty} i^n \left\{ \begin{array}{l} \left[\begin{array}{l} Ce_n(\xi\ ,q) - Me_n(\xi,q) \\ \times Ce_n'(\xi_0,q)/ \\ Me_n'(\xi_0,q) \end{array} \right] \times \\ \times Ce_n'(\eta,q)ce_n(\theta,q) + \\ +i\left[\begin{array}{l} Se_{n+1}(\xi\ ,q) - \\ Ne_{n+1}(\xi,q) \\ \times Se_{n+1}'(\xi_0,q)/ \\ Ne_{n+1}'(\xi_0,q) \end{array} \right] \\ \times se_{n+1}'(\eta,q)se_{n+1}(\theta,q) \end{array} \right\} \qquad (11)$$

We obtain numerical results for the case $\theta = 0^0$ and $\xi = \xi_0 = 0$, that is, an ellipse, for the case where there is a crack. Let the incident load wave be as shown below.

$$u_{\theta 0} = \sum_{n=0}^{\infty} i^n ce_n(\eta,q)ce_n(0,q)/ce_n(\pi/2,q).$$

Numerical results and their analysis.

Numerical results are obtained for the values of the semi-axes of the ellipse from 0,005 to 0,80 ($\delta = b/a$) and for different values θ and k_s. In the calculations for the relaxation of the nucleus, the Rjanitsyn-Koltunov kernel was used. $R(t) = \dfrac{Ae^{-\beta t}}{t^{1-\alpha}}$ [9] with a three-parameter weak singularity, where A, α, β - parameters characterizing the rheological properties of materials [10].

For the Rjanitsy n-Koltunov kernel $A = 0,048;\ \ \beta = 0,05;\ \ \alpha = 0,1$. The results are presented in Tables 1, 2 and 3. Tables 1 and 2 show the changes. $(\sigma_{\eta x_3})_{\max} = \left|\sigma_{\eta x_3} / 2\mu_0 A_p\right|$ ($k_s = 1.7$) and addiction δ and θ .

Table 1. Change in shear stress depending on the ratio of semi-axes

θ	$(\sigma_{\eta x_3})_{\max}$		
	δ		
	0.005	0.50	0.80
0	0.8521	1.0271	1.2632
$\pi/3$	0.9727	1.0193	1.6984
$\pi/2$	0.5938	0.7048	1.2987

Change $\mathrm{Re}(\sigma_{\eta x_3})$ depending on δ =0.15 и θ , in case $\xi \to 0, \eta \to 0$ is given in

Table 2. Change in the real part of the shear stress depending on the angle

r/π	θ		
	0	$\pi/3$	$\pi/2$
0.10	0.0000	0.2951	0.4954
0.20	0.4893	0.0938	0.1962
0.30	0.8132	0.5617	1.0716
0.40	0.0148	0.7983	0.2733
0.50	0.8439	0.0274	0.0928
0.70	0.5761	0.5867	0.1867

The results show that the stresses on the falling side of the ellipse are 20% greater than on the dark side.

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