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Issue Article

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MAXIMUM FALL DISTANCE AND VELOCITY OF A ROCK BLOCK UNDER EXOGENOUS PROCESSES IN A RAILWAY

Abstract: *This article explores the dynamics of a stone block falling onto a railway track due to exogenous factors. In the article, the movement of a falling stone is studied using classical mechanics methods. A mathematical expression for the maximum distance of stone falling is determined from the equation of the law of stone motion. An equation that describes the change in mechanical energy during the rolling of a homogeneous stone is derived. From this equation, an analytical expression for the velocity of a rock block depending on its position is obtained. The problems are solved using the principles of theoretical mechanics. Calculations are conducted and the results are presented for the analytical expression obtained for the velocity of a rock block.*

Key words: *stonefall, railway, stone fall distance, stone velocity, potential energy, kinetic energy.*

Language: *English*

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Introduction

Studying the process of stones falling into the railway track is useful when designing structures to protect against exogenous phenomena. The results of theoretical studies of the rockfall on the railway track can be used in designing and developing barrier schemes to protect railways from exogenous processes.

Modeling is one of the ways to determine the kinematic characteristics of a rock fall, and it is conducted in different ways.

Probabilistic approaches are used to model the trajectory of rock blocks during a rockfall considering the criteria of systematic probabilistic modeling [1]. Spatially distributed modeling of rockfalls in three-dimensional topography was conducted based on a kinematic algorithm [2]. To accurately predict and develop landslide protection measures in mountainous regions, the researchers tested the application of their modeling at the local level comparing the results with existing experimental data.

Rockfall is one of the hindrances to the safe operation of transport in some areas of Tibet. By analyzing and modeling the rockfall process for a certain region at the local level, the impact zone, arrival point, trajectory, height, and kinetic energy of fallen stones were determined [3].

The analysis and forecast of rockfalls were conducted using the developed program for the mathematical model, which predicts the trajectories of the fall and the corresponding parameters for planning repair work [4].

A brief review shows that rockfall studies were conducted using probabilistic approaches and creating software products. The rockfall process can be studied by simple methods of classical mechanics [5, 6]. Analytical expressions for the maximum fall distance and velocity of stone blocks are obtained in the article, using classical mechanics principles.

Material

When designing and developing barrier schemes to protect railway tracks from falling stones, it is necessary to determine the kinematic characteristics of a block of rock. We study the movement of the rock block down a mountain slope during a rockfall using classical mechanics methods.

Consider the movement of a rock block down the domain of an inclined plane when the slope is not bounded by a protective net (Fig. 1). We need to determine the maximum distance and velocity of the stone's fall.

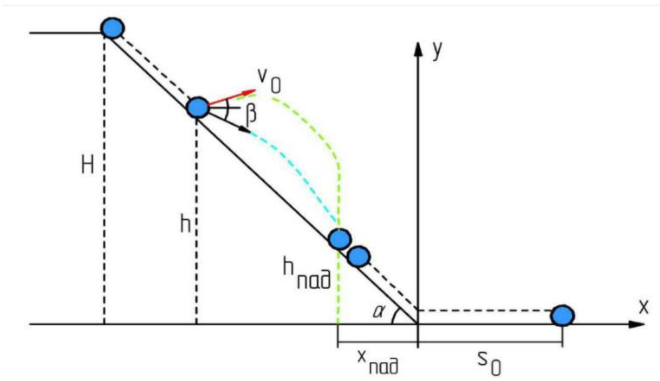

Fig. 1. Scheme of a falling stone motion

1. Determine the maximum distance of the stone's fall. We derive the equation of stone motion based on modeling its free fall [5, 6, 7]. To obtain the equation of motion of a rock on an inclined plane, we construct the equation of projections of forces along the x and *y*-axes: $m\ddot{x} = 0,\ m\ddot{y} = -mg$. Here g - is the acceleration of free fall. Integrating the first equation, we obtain $x = v_0 \cos\beta t - h/tg\alpha,$ here, expression $v_0 \cos\beta t$ is formed by integrating the first equation, and the second expression $h/tg\alpha$ - by moving the origin of the *x*-axis. Integrating the second equation, we obtain $\dot{y} = -gt + c_3$, and integrating it a second time, we obtain $y = -gt^2/2 + c_3 t + c_4$. To determine the integration constants c_3 and c_4, we use the initial conditions at $t = 0$. Substituting the found integration constants, we obtain expression $y = h + v_0 \cos\beta t - gt^2/2.$

Thus, we obtained the following equations of the rock block motion on an inclined plane:

$$x = v_0 \cos\beta t - h/tg\alpha,$$
$$y = h \pm v_0 \sin\beta t - gt^2/2. \quad (1)$$

here β - is the angle of block flight at the point of impact, and the sign "$\pm$" corresponds to the sign of the velocity projection relative to the vertical axis (up or down).

We determine the point where the rock block falls on the horizontal area. Eliminating time from the equation of the law of motion (1) and assuming that $y = 0$, we obtain the following equation for the distance the stone falls:

$$x_{пад} = -h/tg\alpha + \frac{v_0^2\cos\beta}{g}\left[\sqrt{\sin^2\beta + 2gh/v_0^2} \pm \sin\beta\right]; \;\; (x_{пад} > 0). \qquad (2)$$

Using the expression obtained, we find the value of the maximum distance of the stone's fall (for all flight angles):

$$x_{пад}^{\max} = -h/tg\alpha + v_0^2/g\sqrt{1 - \frac{1}{2\left(1+gh/v_0^2\right)}}\left[\frac{1}{\sqrt{2\left(1+gh/v_0^2\right)}} \pm \sqrt{2gh/v_0^2 + \frac{1}{2\left(1+gh/v_0^2\right)}}\right], \; (x_{пад}^{\max} > 0)\,. \; (3)$$

2. We determine the velocity of the stone's fall as it moves along the section of an inclined plane. Such motion is similar to ball rolling along an inclined plane. Let us derive an equation for the change in mechanical energy of the stone block rolling. We assume that the change in mechanical energy equals the work of rolling friction [5, 6, 7]. We consider the stone to be homogeneous. The potential energy of this stone at the top of the rock, i.e. in the initial position where the kinetic energy is zero, is mgH, which expresses the total potential energy of the stone. Here m is the mass of the rock block, g is the acceleration due to gravity, H is the height of the rock, mgh is the potential energy of the rock block at an arbitrary point on the inclined plane, where h is the height of an arbitrary location of the piece of rock on the slope. The kinetic energy of a rock block at an arbitrary point on the slope is $\frac{mv^2}{2} + \frac{J\omega^2}{2}$, where J - is the moment of inertia relative to the axis passing through the center of mass perpendicular to the plane of motion, $\omega = v/r$ is the angular velocity of rotation of the piece of rock (a rock fragment) relative to the instantaneous center of rotation.

Since the rolling friction force is $F_r = f_r N = f_r mg\cos\alpha$, the work of the rolling resistance moment is:

$$A_r = F_r s = f_r mg\cos\alpha(H-h)/\sin\alpha = f_r mgctg\alpha(H-h),$$

where - f_r is the coefficient of rolling friction, and α is the angle of inclination of the inclined plane.

Thus, the equation for the change in mechanical energy of the process of a rock piece rolling down an inclined plane has the following form:

$$mgH - (mgh + \frac{mv^2}{2} + \frac{J\omega^2}{2}) = f_r mgctg\alpha(H-h). \qquad (4)$$

From equation (4), we can determine the rock block velocity at any point on the slope:

$$v = \sqrt{\frac{2g(H-h)}{(1+J/mr^2)}[1 - f_r ctg\alpha]}\,. \qquad (5)$$

If the piece of rock has the shape of a sphere, then the value of expression J/mr^2 in equation (5) is $J/mr^2 = 2/5$ [5] and for this case, the expression for the rock piece velocity takes the following form:

$$v = \sqrt{\frac{2g(H-h)}{(1+2/5)}[1 - f_r ctg\alpha]}\,. \qquad (6)$$

Results

Calculations were performed based on the analytical expression obtained for the rock block velocity depending on its position. Figure 2 shows the calculation results for $H = 10$, $f_r = 0,5$ at inclination angles $\alpha = 30^0$, $\alpha = 45^0$.

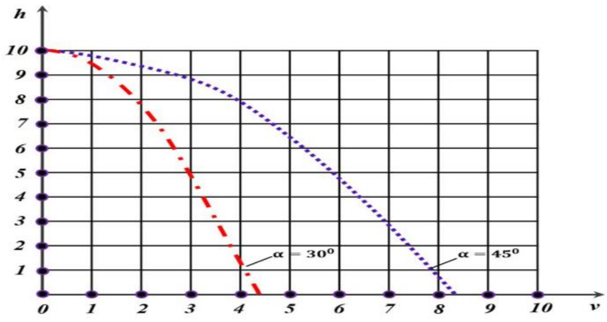

Fig. 2. Change in the velocity of a rock block by height for $H = 10$ m, $f_r = 0,5$ at inclination angles $\alpha = 30^0$, $\alpha = 45^0$.

As seen from the figures, the changes in the velocity of a rock block for different angles of inclination have the same pattern but differ in values. It is evident from Figure 2 that these values at high coefficients of rolling friction significantly vary at different angles of inclination.

Figure 3 shows the change in the velocity of a piece of rock by height for *H*=10 m, f_r=0.04 at angles of inclination $\alpha = 30^0$, $\alpha = 60^0$. Note that the difference between the velocity values of a piece of rock at different angles of inclination at low coefficients of rolling friction is small. This can be explained by the physical meaning of the coefficient of rolling friction, which expresses the state of the geological material of the slope.

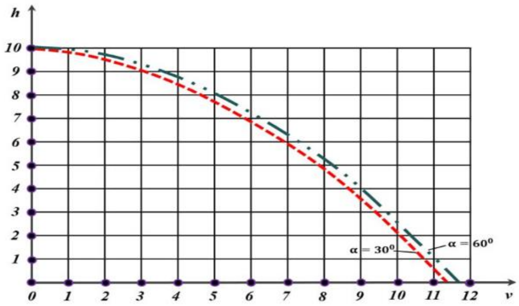

Fig. 3. Change in the velocity of a rock block by height for *H*=10 m, f_r=0.04 at angles of inclination $\alpha = 30^0$, $\alpha = 60^0$.

Conclusion

The maximum distance of a stone fall is determined by the principles of stone motion.

From the analysis of mechanical energy changes in a stone rolling down an inclined plane, we derived an equation that expresses the velocity of the rock fragment depending on its position during a collapse or landslide.

The velocities of the rock block vary at different angles of inclination. However, while the overall pattern remains consistent, the actual values differ significantly, especially at high coefficients of rolling friction. In contrast, at low coefficients of rolling friction, the differences in velocity at various angles of inclination are minimal.

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