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Issue Article

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RESEARCH ON THE LOGISTIC MODEL OF THE POPULATION IN THE MAPLE ENVIRONMENT

***Abstract**: Mathematical models, based on the theory of differential equations, are utilised to describe the dynamics of ecological systems. Distinctive features of such models include the interactions of individual ecosystem elements, as well as the interactions of these elements with external environmental factors in which each element operates. The advent of a contemporary field within mathematics, namely computer mathematics, has engendered an enhanced capacity to undertake ecosystem research at an elevated level. The present article describes an investigation into a logistic model in the Maple computer mathematics system.*

***Key words**: population dynamics, individual, stationary solution, differential equation, mathematical model, Maple.*

***Language**: English*

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Introduction

The mathematical formalisation of ecological objects and their developmental patterns poses a significant challenge, primarily due to the heterogeneity inherent in their internal structures. Consequently, they are regarded as complex systems that function in accordance with the laws of physics, chemistry, and biology. Consequently, these objects are contingent on the influence of random processes that arise due to their complex biochemical composition and multicomponent nature. The development of mathematical models that adequately describe ecological structures is influenced by a number of factors, which significantly complicates their construction. The theory of dynamic equations has found the greatest application in the field of ecology, allowing the description of the temporal dynamics of individual elements, ecological systems as a whole, and their interactions.[1]

One of the objects of study in ecology is populations of living organisms. Point models are utilised for the mathematical modelling of such objects. This approach entails the population being regarded as a unified entity and geometrically represented as a point, with specific physical parameters that delineate the population being assigned to this point. For instance, N – denotes the number of individuals in the population. The point representing the population is then placed on the ON axis. The ON axis is thus representative of the space of population size, with the coordinate of the point denoting the number of individuals in the population.[1-3]

It is evident that the number N is contingent upon time t, given that the population size fluctuates over time for a multitude of reasons. Physical dimensions: The first number of the sequence is

$[N] = axis$ *(individual)*, and the second is $[t] =$ *month (month)*. The sequence is monthly. In the context of many populations, the preference for the terms 'day' and 'minute' is more pronounced. It is evident that such a model fails to consider individual properties and their spatial distribution.

In actuality, the function $N(t)$ accepts integer values. However, in order to construct a differential model, the smoothing principle is employed in the simulation. Consequently, the function $N(t)$ is regarded as continuously differentiable with respect to the time parameter t.

A differential population model that describes the dynamics of a population's size over time is represented as a first-order ordinary differential equation, termed a dynamic equation.:

$$\frac{dN(t)}{dt} = k(t)N(t), \qquad (1)$$

where $k(t) = \alpha(t) - \beta(t)$, $\alpha(t)$ – fertility rate, $\beta(t)$ – destruction coefficient.

A population model described by an equation of the form (1) is referred to as the Malthusian model. The Malthusian model posits that the rate of population growth is directly proportional to the population's current size. The Malthusian model is linear in nature, as evidenced by the linearity of equation (1).[4]

The Malthusian model finds application in the mathematical modelling of population dynamics over brief time periods. The application of the Malthusian model over extended periods of time results in a marked increase in population size, exhibiting exponential growth. However, in reality, this does not occur, since there is some limitation of food resources in the external environment. aking this into account, we define the extinction coefficient as a linear function of population size $N : \beta = \gamma \cdot N(t)$, $\gamma > 0$, where γ is the coefficient of intraspecific competition.[5].

This clarification is deemed to be a natural consequence of the preceding considerations. As the population of individuals with the identifier N increases, so too does the level of competition between said individuals for food. Consequently, the extinction coefficient β experiences an increase. Accordingly, the following equation is obtained:

$$\frac{dN}{dt} = (\alpha - \gamma N) \cdot N, \qquad (2)$$

which is called the logistic equation (the Verhulst–Pearl equation). Equation (2) represents a model of population size change over time and is called the logistic population model. The model has the following formulation: A community of individuals of the same species (population) exists under favorable conditions and at time $t_0 = 0$ has a population size of N_0.. At each subsequent time point, the rate of increase N_0 is proportional to the existing one. The emerging phenomena of extinction due to intraspecific competition reduce the population size proportional to the square of the existing one. [5-6]

Equation (2) has two stationary points $N_1 = 0$, $N_p = \frac{\alpha}{\gamma}$. When examining stability, we have: point N_1 is unstable, point N_p is stable. For a complete study of the logistic model, it is necessary to solve the Cauchy problem. [7]:

$$\frac{dN}{dt} = \alpha\left(1 - \frac{N}{N_p}\right)N, \quad N\big|_{t=0} = N_0.$$

The solution is determined by the logistic curve:

$$N(t) = \frac{N_p N_0 e^{\alpha t}}{N_p - N_0\left(1 - e^{\alpha t}\right)}.$$

An analysis of the solution shows that, regardless of the initial population size, over time the number of individuals tends toward the equilibrium population size of N_p. The number N_p is also called the carrying capacity.

Let's consider a study of the population dynamics model in the Maple system. In the first step, we perform an analytical solution of the model equation (2). For this, we use the DEtools package, which deals with solutions of differential equations. [8-10] We introduce the model equation in the form:

$$\frac{dN}{dt} = \alpha \cdot N - \beta \cdot N^2,$$

in which γ, the coefficient of intraspecific competition in equation (2), was replaced by β. This is done because this variable is reserved in the Maple system. Thus,:

```
restart;
with (DEtools);
eq:=diff(N(t),t)=alphaN(t)-beta(N(t))^2;
```

$$eq := \frac{d}{dt} \mathrm{N}(t) = \alpha\, \mathrm{N}(t) - \beta\, \mathrm{N}(t)^2$$

The model equation is an ordinary differential equation of the order: an equation with separable variables. Its type and autonomy are determined using the commands *odeadvisor* and

```
odeadvisor(eq),autonomous(eq,N(t),t);
```

$$[\_quadrature], true$$

The obtained result indicates that the analytical solution of the equation and analysis can be carried out using the command *dsolve*.

In order to obtain a solution in the form of a Cauchy problem, it is necessary to set the initial conditions: $N(t_0) = N_0$:

```
ny:=N(0)=N[0];
```

$$ny := \mathrm{N}(0) = N_0$$

By applying the *dsolve* command to the model equation and taking into account the initial conditions, we obtain a particular solution of the equation:

```
req:=dsolve({eq,ny},N(t));
```

$$req := \mathrm{N}(t) = -\frac{\alpha\, N_0}{\mathbf{e}^{(-\alpha\, t)} N_0\, \beta - \mathbf{e}^{(-\alpha\, t)} \alpha - N_0\, \beta}$$

After simplifying the last expression, we obtain the dependence of the population size N on time t:

$$Yreq := \mathrm{N}(t) = \frac{\alpha\, N_0\, \mathbf{e}^{(\alpha\, t)}}{N_0\, \beta\, \mathbf{e}^{(\alpha\, t)} - N_0\, \beta + \alpha}$$

Using the command *odetest*[8-10]:

```
odetest(Yreq,eq);
```

$$0$$

the solution is being checked.

In the second stage, we will conduct a mathematical study of the solution to equation $Yreq$. We examine the solution to the equation for continuity using the *discont* command in domain $[0,+\infty)$ from the *readlib(discont)* library. When executing the command, Maple records discontinuity points of the 1st kind on the right-hand side of the desired solution. When finding singular points, it records discontinuity points of the 2 nd kind. [8-10]:

```
alpha:='alpha':beta:='beta':N[0]:='N[0]':
rYreq:=rhs(Yreq);
iscont(rYreq,t=0..infinity,'closed');
tr1:=discont(rYreq,t);
tr2:=singular(rYreq,t);
```

$$rYreq := \frac{\alpha\, N_0\, \mathbf{e}^{(\alpha\, t)}}{N_0\, \beta\, \mathbf{e}^{(\alpha\, t)} - N_0\, \beta + \alpha}$$

FAIL

$$tr1 := \left\{ \frac{\ln\left(-\frac{-\beta N_0 + \alpha}{N_0 \beta}\right) + 2 I \pi \_Z2\sim}{\alpha} \right\}$$

$$tr2 := \left\{ t = \frac{\ln\left(-\frac{-\beta N_0 + \alpha}{N_0 \beta}\right) + 2 I \pi \_Z3\sim}{\alpha} \right\}$$

The moment in time t when the population growth rate is maximum is found using the command *solve*:

tv:=solve(diff(rYreq,t$2)=0,t);

$$tv := \frac{\ln\left(\frac{-\beta N_0 + \alpha}{N_0 \beta}\right)}{\alpha}$$

This section addresses the practical issue of when and how much of a population can be removed from the ecosystem without harming it, so that the total removal is maximized. The maximum removal of population N will be achieved if the population is maintained at a level:

Mch:=solve(tv,N[0]);

$$Mch := \frac{\alpha}{2\beta}$$

We enter the numerical values of the model parameters: N_0 - initial population size, α - integral coefficient of growth and mortality, β - coefficient of intraspecific competition with calculation accuracy of up to 5 digits:

N0:=0.57:N[0]:=N0:
alpha:=0.97:beta:=0.12:Digits:=5:

The maximum possible population size of N is found using the *Limit* command for the condition of its infinitely long lifespan. [8-10]:

Limit(rYreq,t=infinity):%=value(%);

$$\lim_{t \to \infty} \frac{0.5529\, \mathbf{e}^{(0.97\, t)}}{0.0684\, \mathbf{e}^{(0.97\, t)} + 0.9016} = 8.0833$$

The result is checked using the *maximize* command. To do this, we include the library at the beginning of the program *readlib*(*maximize*):

maximize (rYreq,t=0..infinity,location);

$$8.0835, \{[\{t = \text{Float}(\infty)\}, 8.0835]\}$$

To calculate the minimum population size N, use the *minimize* command from the library. *readlib(minimize):*

minimize (rYreq,t=0..infinity,location);

$$0.57002, \{[\{t = 0.\}, 0.57002]\}$$

When the population size N falls below this critical value due to impacts, population recovery becomes impossible.

The maximum population growth rate is calculated using the command *extrema*:

```
extrema (diff(rYreq,t),{},t,'tmax'), tmax=tv;
```

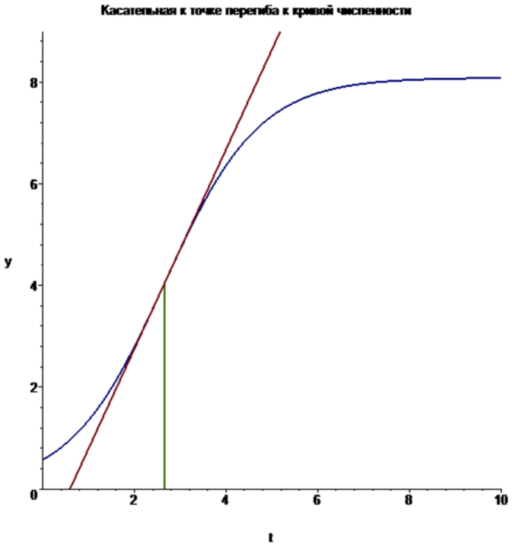
$$\{0.0004, 1.9601\}, \{\{t = 2.6586\}, \{t = 12.462\}\} = 2.6587$$

Under these conditions, the population growth rate will tend to zero:

$$\lim_{t \to \infty} \frac{7555.3\, \mathrm{e}^{(0.97\, t)}}{\left(8.5500\, \mathrm{e}^{(0.97\, t)} + 112.70\right)^2} = 0.$$

The study will be presented in the form of a graphical visualization. The first graph will show the tangent to the inflection point of the population curve. [8-10]:

```
student[showtangent](rYreq,t=tv,t=0..10,y=0..9,
title="Касательная к точке перегиба к кривой численности",thickness=2);
```

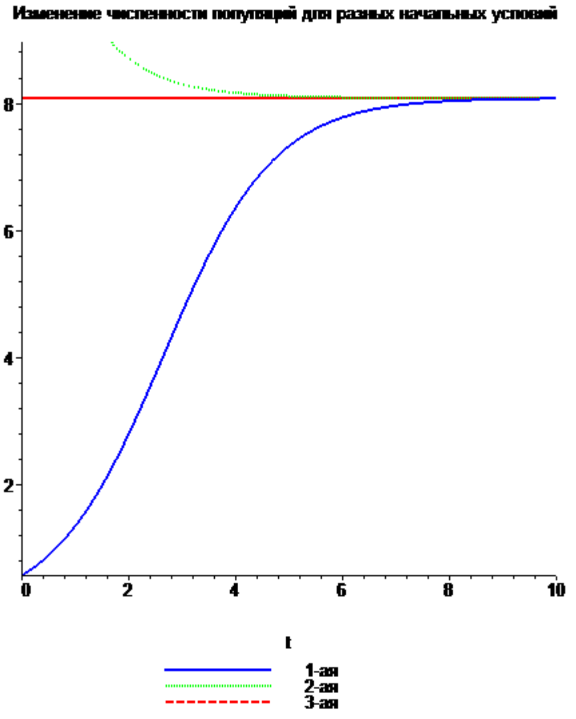


According to the schedule, starting from moment $tv = 2{,}6587$, it is necessary to continuously reduce the population size by N, maintaining the population size no higher than $N = 8{,}0835$. The inflection point has coordinates $\left(\frac{1}{\alpha} \ln \frac{(\alpha - \beta N_0)}{\beta N_0}, \frac{\alpha}{2}\right)$

The second graph shows the change in population size N for different initial conditions.

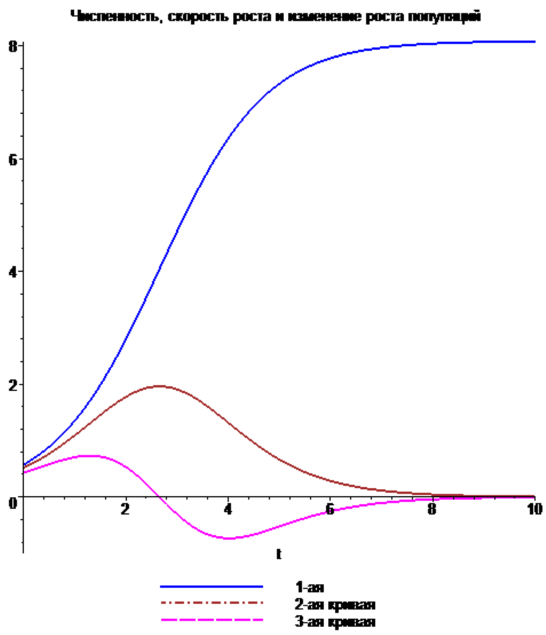
Using the *plot* command, we will display the 1st curve of the solution to the model equation for a given population size of N_0, the 2nd curve of the solution to the model equation with a population size of $N_0 = \frac{\alpha}{\beta}$, at which the population size reduction will be introduced, the 3rd curve of the solution to the model equation with a maximum population size of $N_0 = \frac{\alpha}{2\beta}$, and then we form for the command *display* :

```
N[0]:=N0:G1:=plot(rYreq,t=0..10,linestyle=1,color=blue, legend="1-ая"):
N[0]:=alpha/beta*2:G2:=plot(rYreq,t=0..10,linestyle=2,color=green, legend="2-ая"):
G3:=plot(alpha/beta,t=0..10,linestyle=3,color=red, legend="3-ая"):
display([G1,G2,G3],thickness=2,title="Изменение численности популяций для разных начальных условий");
```



The third graph shows the population size, growth rate, and change in growth rate of the population. [8-10]:

N[0]:=N0:G1:=plot(rYreq,t=0..10,linestyle=1,color=blue, legend="1-ая"):
N[0]:=N0:G4:=plot(diff(rYreq,t),t=0..10,linestyle=4,color=brown, legend="2-ая кривая"):
G5:=plot(diff(rYreq,t$2),t=0..10,y=-1..6,linestyle=5,color=magenta, legend="3-ая кривая"):
display([G1,G4,G5],thickness=2,title="Численность, скорость роста и изменение роста популяций");



The programme that has been developed comprises an analytical solution to the model equations and a mathematical and graphical analysis of the model in the Maple computer mathematics system. The analysis program for population dynamics is automated. This approach enables the acquisition of results while circumventing the necessity for routine calculations and the associated computational complexities.

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