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Issue Article

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ANALYTICAL SOLUTION TO THE PROBLEM OF FLUID FLOW FROM A VERSIERA-SHAPED CHANNEL

Abstract: *This article models fluid flow in a versiera-shaped pipeline. A stationary problem is considered. Analytical expressions for the fluid hydrodynamic parameters are obtained from the continuity equation and by integrating the integral-differential equation for determining the pressure in a channel of arbitrary shape, derived from the Navier-Stokes equation.*

Key words: *fluid, pipeline, analytical solution, versiera.*
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Introduction

Fluid flow from nozzles of various geometries is a pressing problem in fluid mechanics and is of great practical importance. Fluid flow from nozzles commonly found in engineering has been extensively studied and discussed in hydraulics literature [1].

The results of a study on the flow of viscous fluid from a vertically positioned pipe through a circular bottom opening with an adjustable cross-section are presented in reference [2]. Fluid is considered a continuous medium with a center of mass, and it fills the pipeline cavity. Assuming laminar flow of an incompressible viscous fluid, an equation is derived relating the velocity of its center of mass to its height relative to the discharge orifice. Furthermore, an analytical expression is derived describing the dependence of the outlet orifice cross-sectional parameters on the height of a fluid column with varying viscosity. This enables the practical application of regulated fluid discharge techniques to be controlled.

Studying the flow of Newtonian fluids from nozzles of various configurations has become a pressing problem in polymer production. A model of the slow flow of a viscoelastic fluid with a free surface is developed; such a flow occurs as the polymer fluid enters and exits the forming nozzle [3]. The fluid flow is described by the equations of conservation of mass, momentum, and energy, supplemented by the Giesekus rheological equation of state. A robust numerical algorithm for solving the problem is developed using the finite element method. Numerical studies were conducted to determine the shape of the exit jet for various flow regimes and nozzle configurations. The distribution of fluid velocity, pressure, stress, and temperature was examined as the heating of the nozzle wall increased. Numerical results were obtained for the dependence of the effect of polymer swelling on rheological model parameters and temperature factors.

Numerous experiments were conducted on the flow of liquids and gases through nozzles of various configurations. The results of experimental studies on the horizontal flow of heated gas and superheated water through nozzles with circular cross-sections and different outlet cut angles are presented in reference [4]. A comparative analysis of the main characteristics of liquid and gas jets flowing under different initial conditions and through channels of various configurations was conducted. It was established that when heated gas flows through nozzles with a cut, no displacement of the jet axis relative to the outlet axis was observed. A different pattern was observed when superheated water flows through nozzles with a cut. In this case, there is a displacement of the jet axis in the direction of the cut relative to the outlet axis. Practical recommendations were proposed for the application of the discovered patterns in engine engineering (for fuel atomization by injectors).

The study focused on the outflow of liquids and gases from various nozzle configurations, taking chemical reactions into account. Specifically, the behavior of jet flow in a transition nozzle was examined to understand the influence of chemical reaction mechanisms [5]. The simulations utilized Reynolds-averaged Navier-Stokes equations, and the results were consistent with experimentally obtained pressure ratio values in the transition nozzle.

In this article, we derive analytical expressions for the hydrodynamic parameters of a fluid flowing through a pipeline, which is geometrically described as a versiera.

Problem statement

We consider the flow of a fluid in a channel, the shape of which is described by the following formula:

$$x(a^2+y^2)=a^3, \qquad (1)$$

the geometric description of this channel is a versiera (Fig. 1).

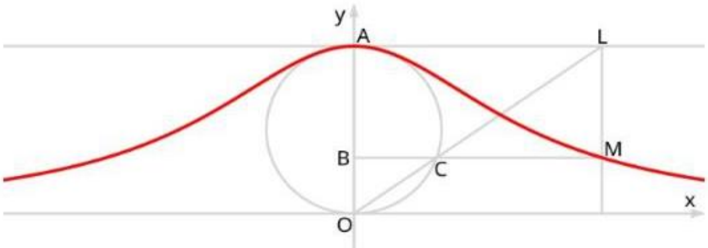

Fig. 1. Versiera

To determine the separate representation of the change and geometric arrangement of the x and y coordinates, we modify formula (1):

$$y(a^2+x^2)=a^3.$$

Interchanging the x and y coordinates yields:

$$-y(a^2+x^2)=a^3.$$

Next, changing the sign of the y coordinate yields:

$$(a-y)(a^2+x^2)=a^3$$

and moving it by a height a, for the resulting curve we obtain:

$$(a+b-y)(a^2+x^2)=a^3.$$

Moving it by distance ±c from point x = 0, and placing a circular cylindrical channel at this distance, we obtain the following curve:

$$(a+b-y)\left[a^2+(|x|-c)^2\right]=a^3 \quad |x|\geq c \qquad (2)$$

$$y=b \qquad |x|\leq c,$$

which describes the shape of the channel along which the fluid flows.

To determine the value of a, we take into account that in the channel inlet section x=-x_{-}, R=R_{-}, and from (3.2) we obtain:

$$R=a+b-\frac{a^3}{a^2+(x-c)^2}. \qquad (3)$$

Multiplying both sides of (3.3) by

$$a^2+(x-c)^2,$$

we have

$$R(a^2+(x-c)^2)=a(x-c)+a^2b=b(x-c)^2,$$

hence

$$(R-b)a^2-(x-c)^2a+(R-b)(x-c)^2=0.$$

Hence, a quadratic equation is formed to determine the value of a:

$$a^2-\frac{x-c+\sqrt{(x-c)^2-4(R-b)^2}}{2(R-b)}(x-c). \qquad (4)$$

Problem solution

To determine the hydrodynamic parameters of a channel, we use the following formulas for determining the average flow rate and pressure as a fluid flows out of a channel of arbitrary shape [6]:

$$P=\rho U(U_{-}-U+8\nu\int_{-x}^{x}\frac{dx}{R^2}),$$

$$U=(\frac{R}{R_{-}})^2U_{-}.$$

To calculate the integral for a given channel, we define the integrand as a simple fraction:

$$\frac{1}{R^2}=\frac{1}{a+b-\dfrac{a^3}{a^2+(|x|-c)^2}}=\frac{a^2+(|x|-c)^2-\dfrac{a^3}{a+b}+\dfrac{a^3}{a+b}}{(a+b)\left[a^2+(|x|-c)^2\right]-a^3}=$$

$$=\frac{1}{a+b}+\frac{a^3}{(a+b)^2}\frac{1}{a^2+(|x|-c)^2-\dfrac{a^3}{a+b}}=\frac{1}{a+b}+\frac{a^3}{a+b}\frac{1}{(|x|-c)^2+a^2b}.$$

Since at the initial cross-section of the channel $|x|$=-x, and the integral has the value of

$$\int_{-x}^{x}(\frac{1}{a+b}+\frac{a^3}{a+b}\frac{1}{(x+c)^2+a^2b})dx=\frac{x}{a+b}\int_{x_{-}}^{x}+\frac{a^3}{a+b}\int_{x_{-}}^{x}\frac{dx}{(x+c)^2+a^2b}=$$

$$=\frac{x-x_{-}}{a+b}+\frac{a^2}{\sqrt{b}(a+b)}(arctg\frac{x+c}{a\sqrt{b}}-arctg\frac{x_{-}+c}{a\sqrt{b}})$$

at the end of the inlet curved section, the average flow rate and pressure are determined as follows:

$$U_1=\left(\frac{R_{-}}{b}\right)^2U_{-}$$

$$P_1=\rho U_1\left[U_{-}-U_1+8\nu\left(\frac{x_{-}+c}{a+b}+\frac{a^2}{\sqrt{b}(a+b)}arctg\frac{x_{-}+c}{a\sqrt{b}}\right)\right]. \qquad (5)$$

The average flow rate and pressure in the fluid flow zone are determined by the arithmetic expression of a versiera (1).

Conclusion

Analytical expressions for the hydrodynamic parameters of a fluid flowing out of a channel, which is geometrically described by a versiera, have been derived. The resulting solution can be used to calculate the fluid flow parameters in channels with this specific geometry.

References:

1. Frenkel, N.Z. (2014). *Hydraulics*. Textbook. "Book on Demand". 2014. 451 p.
2. Kumitsky, B.M., Evsikova, N.Yu., & Savrasova, N.A. (2024). Viscous Liquid Flow from the Adjustable Bottom Orifice of a Vertical Pipeline. Proceedings of the Conference «*Physical Foundations of High Technologies».* 2024. pp. 36-43. doi: 10.58168/PBST_36-43.
3. Snigerev, B.A. (2011). Mathematical Modeling of Non-isothermal Extrusion of Polymer Liquid from a Stepped Forming Nozzle. *Journal of Higher Educational Institutions. Volga Region. Technical Sciences*. №1. 2011. pp. 196-201.
4. Zhilkin, B.P., Plotnikov, L.V., et al. (2019). «The influence of the outlet angle of a cylindrical channel on the formation of jets of heated gas and superheated liquid» *High Temperature Thermophysics*, 2019, Vol. 57, Issue 3, pp. 431-436. https://doi.org/10.1134/S0040364419030220
5. Dirk, Sch., Chloé, G., Ralf, S., Michael, O., Sebastian, K., & Volker, H. (2017). *Numerical Model for Nozzle Flow Application Under Liquid Oxygen/Methane Hot-Flow Conditions*. Aerospace Research Central. Published Online: 6 Sep 2017. Retrieved from https://doi.org/10.2514/1.B36611
6. Karimov, K., Khudjaev, M., & Akhmedov, A. (2021). Modeling fluid outflow from a channel consisting of three different segments. *Journal "E3S Web of Conferences".* 258, 08021 (2021). https://doi.org/10.1051/e3sconf/202125808021